

Fermion Masses and Flavor Mixings in a Model with S_4 Flavor Symmetry

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Abstract

We present a supersymmetric model of quark and lepton based on $S_4 \times Z_3 \times Z_4$ flavor symmetry. The S_4 symmetry is broken down to Klein four and Z_3 subgroups in the neutrino and the charged lepton sectors respectively. Tri-Bimaximal mixing and the charged lepton mass hierarchies are reproduced simultaneously at leading order. Moreover, a realistic pattern of quark masses and mixing angles is generated with the exception of the mixing angle between the first two generations, which requires a small accidental enhancement. It is remarkable that the mass hierarchies are controlled by the spontaneous breaking of flavor symmetry in our model. The next to leading order contributions are studied, all the fermion masses and mixing angles receive corrections of relative order λ_c^2 with respect to the leading order results. The phenomenological consequences of the model are analyzed, the neutrino mass spectrum can be normal hierarchy or inverted hierarchy, and the combined measurement of the $0\nu 2\beta$ decay effective mass $m_{\beta\beta}$ and the lightest neutrino mass can distinguish the normal hierarchy from the inverted hierarchy.

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1 Introduction

Neutrino has provided us a good window to the new physics beyond the Standard Model. Neutrino oscillation experiments have provided solid evidence that neutrinos have small but non-zero masses. Global data fit to the current neutrino oscillation data demonstrates that the mixing pattern in the leptonic sector is so different from the one in the quark sector. Two independent fits for the mixing angles and the mass squared differences are listed in Table 1.

parameter	Ref. [1]		Ref. [2]	
	best fit $\pm 1\sigma$	3σ interval	best fit $\pm 1\sigma$	3σ interval
Δm_{21}^2 [10^{-5}eV^2]	$7.65^{+0.23}_{-0.20}$	7.05–8.34	$7.67^{+0.22}_{-0.21}$	7.07–8.34
Δm_{31}^2 [10^{-3}eV^2]	$\pm 2.40^{+0.12}_{-0.11}$	$\pm(2.07\text{--}2.75)$	-2.39 ± 0.12 $+2.49 \pm 0.12$	$-(2.02\text{--}2.79)$ $+(2.13\text{--}2.88)$
$\sin^2 \theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.25–0.37	$0.321^{+0.023}_{-0.022}$	0.26–0.40
$\sin^2 \theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.36–0.67	$0.47^{+0.07}_{-0.06}$	0.33–0.64
$\sin^2 \theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.056	0.003 ± 0.015	≤ 0.049

Table 1: Three flavour neutrino oscillation parameters from two global data fits [1, 2].

As is obvious, the current neutrino oscillation data is remarkably compatible with the so called Tri-Bimaximal (TB) mixing pattern [3], which suggests the following mixing pattern

$$\sin^2 \theta_{12,TB} = \frac{1}{3}, \quad \sin^2 \theta_{23,TB} = \frac{1}{2}, \quad \sin^2 \theta_{13,TB} = 0 \quad (1)$$

These values lie in the 1σ range of global data analysis shown in Table 1². Correspondingly, the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix is given by

$$U_{PMNS}^{TB} = U_{TB} \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) \quad (2)$$

where α_{21} and α_{31} are the Majorana CP violating phases, and U_{TB} is given by

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

The mixing in the quark sector is described by the famous CKM matrix [4], and there is large mass hierarchies within the quarks and charged leptons sectors respectively [5]. The origin of the observed fermion mass hierarchies and flavor mixings is a great puzzle in particle physics. Nowadays promising candidates for understanding such issue are the models based on spontaneously breaking flavor symmetry, various models based on discrete or continuous flavor symmetry have been proposed so far [6, 7]. Recently it was found that flavor symmetry based on discrete group is particularly suitable to reproduce specific mixing pattern at leading order

² $\sin^2 \theta_{12,TB}$ is exactly within the 1σ range of the second global data fit, whereas it slightly above the 1σ up limit of the first fit.

[8]. The A_4 models are especially attractive, it has received considerable interest in the recent past [9–14]. So far various A_4 flavor models have been proposed, and their phenomenological consequences were analyzed [15–18]. These models assumed that A_4 symmetry is realized at a high energy scale, the lepton fields transform nontrivially under the symmetry group, and the flavor symmetry is spontaneously broken by a set of flavons with the vacuum expectation value (VEV) along a specific direction. The misalignment in the flavor space between the charged lepton and the neutrino sectors results in the TB lepton mixing.

If extend the A_4 symmetry to the quark sector, the quark mixing matrix V_{CKM} turns out to be unity matrix at leading order [11]. However, the subleading contributions of the higher dimensional operators are too small to provide large enough deviations of V_{CKM} from the identity matrix. The possible ways of resolving this issue are to consider new sources of symmetry breaking or enlarge the symmetry group. Two discrete groups T' [19–26] and S_4 [27–32] are found to be promising, both groups have two dimensional irreducible representation, which is very useful to describing the quark sector. The S_4 symmetry is particularly interesting, S_4 as a horizontal symmetry group has been proposed long ago [33], and some models with different purposes have been built [34]. Recently it was claimed to be minimal flavor group capable of yielding the TB mixing without fine tuning [35–37]. However, Grimus et al. were against this point [38].

In this work, we build a SUSY model based on $S_4 \times Z_3 \times Z_4$ flavor group, the neutrino mass is generated via the conventional type I See-Saw mechanism [39]. Our model naturally produces the TB mixing and the charged lepton mass hierarchy at leading order. Furthermore, we extend the model to the quark sector, the realistic patterns of quark masses and mixing angles are generated. In our model the mass hierarchies are controlled by the spontaneous breaking of the flavor symmetry instead of the Froggatt-Nielsen (FN) mechanism [40].

This article is organized as follows. Section 2 is the group theory of S_4 group, where the subgroup, the equivalent class, and the representation of S_4 are presented. In section 3 we justify the vacuum alignment of our model in the supersymmetric limit. In section 4 we present our model in both the lepton and quark sectors, its basic features and theoretical predictions are discussed. In section 5 we analyze the phenomenological implications of the model in detail, which include the mass spectrum, neutrinoless double beta decay and the Majorana CP violating phases etc. The corrections induced by the next to leading order terms are studied in section 6. Finally we summarize our results in the conclusion section.

2 The discrete group S_4

S_4 is the permutation group of 4 objects. The group has 24 distinct elements, and it can be generated by two elements S and T obeying the relations

$$S^4 = T^3 = 1, \quad ST^2S = T \quad (4)$$

Without loss of generality, we could choose

$$S = (1234), \quad T = (123) \quad (5)$$

where the cycle (1234) denotes the permutation $(1, 2, 3, 4) \rightarrow (2, 3, 4, 1)$, and (123) means $(1, 2, 3, 4) \rightarrow (2, 3, 1, 4)$. The 24 elements belong to 5 conjugate classes and are generated

from S and T as follows

$$\begin{aligned}
\mathcal{C}_1 : & 1 \\
\mathcal{C}_2 : & STS^2 = (12), TSTS^2 = (13), ST^2 = (14), S^2TS = (23), TST = (24), T^2S = (34) \\
\mathcal{C}_3 : & TS^2T^2 = (12)(34), S^2 = (13)(24), T^2S^2T = (14)(23) \\
\mathcal{C}_4 : & T = (123), T^2 = (132), T^2S^2 = (124), S^2T = (142), S^2TS^2 = (134), STS = (143), \\
& S^2T^2 = (234), TS^2 = (243) \\
\mathcal{C}_5 : & S = (1234), T^2ST = (1243), ST = (1324), TS = (1342), TST^2 = (1423), S^3 = (1432)
\end{aligned}$$

The structure of the group S_4 is rather rich, it has thirty proper subgroups of orders 1, 2, 3, 4, 6, 8, 12 or 24. Concretely, the subgroups of S_4 are as follows

1. The trivial group only consisting of the unit element.
2. Six two-element subgroups generated by a transposition of the form $\{1, (ij)\}$ with $i \neq j$
 $H_2^{(1)} = \{1, STS^2\}$, $H_2^{(2)} = \{1, TSTS^2\}$, $H_2^{(3)} = \{1, ST^2\}$, $H_2^{(4)} = \{1, S^2TS\}$, $H_2^{(5)} = \{1, TST\}$ and $H_2^{(6)} = \{1, T^2S\}$
3. Three two-element subgroups generated by a double transition of the form $\{1, (ij)(kl)\}$ with $i \neq j \neq k \neq l$
 $H_2^{(7)} = \{1, TS^2T^2\}$, $H_2^{(8)} = \{1, S^2\}$, $H_2^{(9)} = \{1, T^2S^2T\}$
4. Four subgroups of order three, which is spanned by a three-cycle
 $H_3^{(1)} = \{1, T, T^2\}$, $H_3^{(2)} = \{1, T^2S^2, S^2T\}$, $H_3^{(3)} = \{1, S^2TS^2, STS\}$, $H_3^{(4)} = \{1, S^2T^2, TS^2\}$
5. The four-element subgroups generated by a four-cycle, they are of the form $\{1, g, g^2, g^3\}$ with g any four-cycle
 $H_4^{(1)} = \{1, S, S^2, S^3\}$, $H_4^{(2)} = \{1, TS, T^2ST, T^2S^2T\}$, $H_4^{(3)} = \{1, TST^2, ST, TS^2T^2\}$
6. The four-element subgroups generated by two disjoint transpositions, which is isomorphic to Klein four group
 $H_4^{(4)} = \{1, STS^2, T^2S, TS^2T^2\}$, $H_4^{(5)} = \{1, TSTS^2, TST, S^2\}$, $H_4^{(6)} = \{1, ST^2, S^2TS, T^2S^2T\}$
7. The order four subgroup comprising of the identity and three double transitions, which is isomorphic to Klein four group
 $H_4^{(7)} = \{1, TS^2T^2, S^2, T^2S^2T\}$
8. Four subgroups of order six, which is isomorphic to S_3 . They are the permutation groups of any three of the four objects, leaving the fourth invariant
 $H_6^{(1)} = \{1, STS^2, TSTS^2, S^2TS, T, T^2\}$, $H_6^{(2)} = \{1, STS^2, ST^2, TST, T^2S^2, S^2T\}$,
 $H_6^{(3)} = \{1, TSTS^2, ST^2, T^2S, S^2TS^2, STS\}$, $H_6^{(4)} = \{1, S^2TS, TST, T^2S, S^2T^2, TS^2\}$

	classes				
	\mathcal{C}_1	\mathcal{C}_2	\mathcal{C}_3	\mathcal{C}_4	\mathcal{C}_5
$n_{\mathcal{C}_i}$	1	6	3	8	6
$h_{\mathcal{C}_i}$	1	2	2	3	4
1_1	1	1	1	1	1
1_2	1	-1	1	1	-1
2	2	0	2	-1	0
3_1	3	1	-1	0	-1
3_2	3	-1	-1	0	1

Table 2: Character table of the S_4 group. $n_{\mathcal{C}_i}$ denotes the number of the elements contained in the class \mathcal{C}_i , and $h_{\mathcal{C}_i}$ is the order of the elements of \mathcal{C}_i .

9. Three eight-element subgroups, which is isomorphic to D_4

$$\begin{aligned} H_8^{(1)} &= \{1, TSTS^2, TST, S, S^3, TS^2T^2, S^2, T^2S^2T\}, \\ H_8^{(2)} &= \{1, STS^2, T^2S, ST, TST^2, TS^2T^2, S^2, T^2S^2T\} \\ H_8^{(3)} &= \{1, ST^2, S^2TS, T^2ST, TS, TS^2T^2, S^2, T^2S^2T\} \end{aligned}$$

10. The alternating group A_4

$$A_4 = \{1, TS^2T^2, S^2, T^2S^2T, T, T^2, T^2S^2, S^2T, S^2TS^2, STS, S^2T^2, TS^2\}$$

11. The whole group

In particular, $H_4^{(7)}$ and A_4 are the invariant subgroups of S_4 . Since the number of the un-equivalent irreducible representation is equal to the number of class, the S_4 group has five irreducible representations: 1_1 , 1_2 , 2 , 3_1 and 3_2 . 1_1 is the identity representation and 1_2 is the antisymmetric one. The Young diagram for the two dimensional representations is self associated, and the Young diagrams corresponding to the three dimensional representations 3_1 and 3_2 are associated Young diagrams. For the same group element, the representation matrices of 3_1 and 3_2 are exactly the same if the element is an even permutation. Whereas the overall signs are opposite if the group element is an odd permutation. It is notable that S_4 together with T' is the smallest group containing one, two and three dimensional representations. The character table of S_4 group is shown in Table 2.

From the character table of the S_4 group, we can straightforwardly obtain the multiplication rules between the various representations

$$\begin{aligned} 1_i \otimes 1_j &= 1_{((i+j) \bmod 2)+1}, & 1_i \otimes 2 &= 2, & 1_i \otimes 3_j &= 3_{((i+j) \bmod 2)+1} \\ 2 \otimes 2 &= 1_1 \oplus 1_2 \oplus 2, & 2 \otimes 3_i &= 3_1 \oplus 3_2, & 3_i \otimes 3_i &= 1_1 \oplus 2 \oplus 3_1 \oplus 3_2, \\ 3_1 \otimes 3_2 &= 1_2 \oplus 2 \oplus 3_2 \oplus 3_2, & \text{with } i, j &= 1, 2 \end{aligned} \tag{6}$$

The explicit representation matrices of the generators S , T and other group elements for the five irreducible representations are listed in Appendix A. From these representation matrices, one can explicitly calculate the Clebsch-Gordan coefficients for the decomposition of the product representations, and the same results as those in Ref. [29] are obtained.

	ℓ	e^c	μ^c	τ^c	ν^c	Q_L	Q_3	u^c	c^c	t^c	d^c	s^c	b^c	$h_{u,d}$	φ	χ	θ	η	ϕ	Δ
S_4	3_1	1_1	1_2	1_1	3_1	2	1_1	1_1	1_2	1_1	1_1	1_2	1_2	1_1	3_1	3_2	1_2	2	3_1	1_2
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	1	1	1	ω	1	1	1	1	1	1	ω^2	ω^2	ω^2
Z_4	1	i	-1	$-i$	1	-1	1	i	1	1	1	1	1	1	i	i	1	1	1	-1

Table 3: The transformation rules of the matter fields and the flavons under the symmetry groups S_4 , Z_3 and Z_4 . ω is the third root of unity, i.e. $\omega = e^{i\frac{2\pi}{3}} = (-1 + i\sqrt{3})/2$. We denote $Q_L = (Q_1, Q_2)^t$ which are doublets of S_4 , where $Q_1 = (u, d)^t$ and $Q_2 = (c, s)^t$ are the electroweak SU(2) doublets of the first two generations. $Q_3 = (t, b)^t$ is the electroweak SU(2) doublet of the third generation.

3 Field content and the vacuum alignment

The model is supersymmetric and based on the discrete symmetry $S_4 \times Z_3 \times Z_4$. Supersymmetry is introduced in order to simplify the discussion of the vacuum alignment. The S_4 component controls the mixing angles, the auxiliary Z_3 symmetry guarantees the misalignment in flavor space between the neutrino and the charged lepton mass eigenstates, and the Z_4 component is crucial to eliminating the unwanted couplings and reproducing the observed mass hierarchy. The fields of the model and their classification under the flavor symmetry are shown in Table 3, where two Higgses doublets $h_{u,d}$ of the minimal supersymmetric standard model are present. If the S_4 flavor symmetry is preserved until the electroweak scale, then all the fermions would be massless. Therefore S_4 symmetry should be broken by the suitable flavon fields, which are standard model singlets. Another critical issue of the flavor model building is the vacuum alignment, a global continuous $U(1)_R$ symmetry is exploited to simplify the vacuum alignment problem. This symmetry is broken to the discrete R parity once we include the gaugino mass in the model. The matter fields carry +1 R-charge, the Higgses and the flavon supermultiplets have R-charge 0. The spontaneous breaking of S_4 symmetry can be implemented by introducing a new set of multiplets, the driving fields carrying 2 unit R-charge. Consequently the driving fields enter linearly into the superpotential. The suitable driving fields and their transformation properties are shown in Table 4. In the following, we will discuss the minimization of the scalar potential in the supersymmetric limit. At the leading order, the most general superpotential dependent on the driving fields, which is invariant under the flavor symmetry group $S_4 \times Z_3 \times Z_4$, is given by

Fields	φ^0	ξ'^0	θ^0	η^0	ϕ^0	Δ^0
S_4	3_1	1_2	1_1	2	3_2	1_1
Z_3	1	1	1	ω^2	ω^2	ω^2
Z_4	-1	-1	1	1	1	1

Table 4: The driving fields and their transformation properties under the flavor group $S_4 \times Z_3 \times Z_4$.

$$w_v = g_1(\varphi^0(\varphi\varphi)_{3_1})_{1_1} + g_2(\varphi^0(\chi\chi)_{3_1})_{1_1} + g_3(\varphi^0(\varphi\chi)_{3_1})_{1_1} + g_4\xi'^0(\varphi\chi)_{1_2} + M_\theta^2\theta^0 + \kappa\theta^0\theta^2 + f_1(\eta^0(\eta\eta)_{2_1})_{1_1} + f_2(\eta^0(\phi\phi)_{2_1})_{1_1} + f_3(\phi^0(\eta\phi)_{3_2})_{1_1} + h_1\Delta^0\Delta^2 + h_2\Delta^0(\eta\eta)_{1_1} + h_3\Delta^0(\phi\phi)_{1_1} \quad (7)$$

where the subscript 1_1 denotes the contraction in 1_1 , similar rule applies to other subscripts 1_2 , 2 , 3_1 and 3_2 . In the SUSY limit, the vacuum configuration is determined by the vanishing of the derivative of w_v with respect to each component of the driving fields

$$\begin{aligned}
\frac{\partial w_v}{\partial \varphi_1^0} &= 2g_1(\varphi_1^2 - \varphi_2\varphi_3) + 2g_2(\chi_1^2 - \chi_2\chi_3) + g_3(\varphi_2\chi_3 - \varphi_3\chi_2) = 0 \\
\frac{\partial w_v}{\partial \varphi_2^0} &= 2g_1(\varphi_2^2 - \varphi_1\varphi_3) + 2g_2(\chi_2^2 - \chi_1\chi_3) + g_3(\varphi_3\chi_1 - \varphi_1\chi_3) = 0 \\
\frac{\partial w_v}{\partial \varphi_3^0} &= 2g_1(\varphi_3^2 - \varphi_1\varphi_2) + 2g_2(\chi_3^2 - \chi_1\chi_2) + g_3(\varphi_1\chi_2 - \varphi_2\chi_1) = 0 \\
\frac{\partial w_v}{\partial \xi'^0} &= g_4(\varphi_1\chi_1 + \varphi_2\chi_3 + \varphi_3\chi_2) = 0 \\
\frac{\partial w_v}{\partial \theta^0} &= M_\theta^2 + \kappa\theta^2 = 0
\end{aligned} \tag{8}$$

This set of equations admit the solution

$$\langle \varphi \rangle = (0, v_\varphi, 0), \quad \langle \chi \rangle = (0, v_\chi, 0), \quad \langle \theta \rangle = v_\theta \tag{9}$$

with

$$v_\varphi^2 = -\frac{g_2}{g_1}v_\chi^2, \quad v_\theta^2 = -\frac{M_\theta^2}{\kappa}, \quad v_\chi \text{ undetermined} \tag{10}$$

From the driving superpotential w_v , we can also derive the equations from which to extract the vacuum expectation values of η , ϕ and Δ

$$\begin{aligned}
\frac{\partial w_v}{\partial \eta_1^0} &= f_1\eta_1^2 + f_2(\phi_3^2 + 2\phi_1\phi_2) = 0 \\
\frac{\partial w_v}{\partial \eta_2^0} &= f_1\eta_2^2 + f_2(\phi_2^2 + 2\phi_1\phi_3) = 0 \\
\frac{\partial w_v}{\partial \phi_1^0} &= f_3(\eta_1\phi_2 - \eta_2\phi_3) = 0 \\
\frac{\partial w_v}{\partial \phi_2^0} &= f_3(\eta_1\phi_1 - \eta_2\phi_2) = 0 \\
\frac{\partial w_v}{\partial \phi_3^0} &= f_3(\eta_1\phi_3 - \eta_2\phi_1) = 0 \\
\frac{\partial w_v}{\partial \Delta^0} &= h_1\Delta^2 + 2h_2\eta_1\eta_2 + h_3(\phi_1^2 + 2\phi_2\phi_3) = 0
\end{aligned} \tag{11}$$

The solution to the above six equations is

$$\langle \eta \rangle = (v_\eta, v_\eta), \quad \langle \phi \rangle = (v_\phi, v_\phi, v_\phi), \quad \langle \Delta \rangle = v_\Delta \tag{12}$$

with the conditions

$$v_\phi^2 = -\frac{f_1}{3f_2}v_\eta^2, \quad v_\Delta^2 = \frac{f_1h_3 - 2f_2h_2}{f_2h_1}v_\eta^2, \quad v_\eta \text{ undetermined} \tag{13}$$

The vacuum expectation values (VEVs) of the flavons can be very large, much larger than the electroweak scale, and we expect that all the VEVs are of a common order of magnitude. This

is a very common assumption in the flavor model building, which guarantees the reasonability of the subsequent perturbative expansion in inverse power of the cutoff scale Λ . Acting on the vacuum configurations of Eq.(9) and Eq.(12) with the elements of the flavor symmetry group S_4 , we can see that the VEVs of η and ϕ are invariant under four elements 1, TST , $TSTS^2$ and S^2 , which exactly constitute the Klein four group $H_4^{(5)}$. On the contrary, the VEVs of φ and χ break S_4 completely. Under the action of T or T^2 , the directions of $\langle\varphi\rangle$ and $\langle\chi\rangle$ are invariant except an overall phase. Considering the enlarged group $S_4 \times Z_3$, the vacuum configuration Eq.(9) preserves the subgroup Z_3 generated by ωT , which is defined as the simultaneous transformation of $T \in S_4$ and $\omega \in Z_3$. As we shall see later that the S_4 flavor symmetry is spontaneously broken down by the VEVs of η and ϕ in the neutrino sector at the leading order(LO), and it is broken down by the VEVs of φ and χ in the charged lepton sector. Whereas both η , ϕ and φ , χ are involved in generating the quark masses. The S_4 flavor symmetry is broken into the Klein four symmetry $H_4^{(5)}$ and the Z_3 symmetry generated by T in the neutrino and the charged lepton sector respectively at LO. This symmetry breaking chain is crucial to generating the TB mixing.

4 The model with $S_4 \times Z_3 \times Z_4$ flavor symmetry

In this section we shall propose a concise supersymmetric (SUSY) model based on $S_4 \times Z_3 \times Z_4$ flavor symmetry with the vacuum alignment of Eq.(9) and Eq.(12).

4.1 Charged leptons

The charged lepton masses are described by the following superpotential

$$\begin{aligned}
w_\ell = & \frac{y_{e1}}{\Lambda^3} e^c (\ell\varphi)_{11} (\varphi\varphi)_{11} h_d + \frac{y_{e2}}{\Lambda^3} e^c ((\ell\varphi)_2 (\varphi\varphi)_2)_{11} h_d + \frac{y_{e3}}{\Lambda^3} e^c ((\ell\varphi)_{31} (\varphi\varphi)_{31})_{11} h_d \\
& + \frac{y_{e4}}{\Lambda^3} e^c ((\ell\chi)_2 (\chi\chi)_2)_{11} h_d + \frac{y_{e5}}{\Lambda^3} e^c ((\ell\chi)_{31} (\chi\chi)_{31})_{11} h_d + \frac{y_{e6}}{\Lambda^3} e^c (\ell\varphi)_{11} (\chi\chi)_{11} h_d \\
& + \frac{y_{e7}}{\Lambda^3} e^c ((\ell\varphi)_2 (\chi\chi)_2)_{11} h_d + \frac{y_{e8}}{\Lambda^3} e^c ((\ell\varphi)_{31} (\chi\chi)_{31})_{11} h_d + \frac{y_{e9}}{\Lambda^3} e^c ((\ell\chi)_2 (\varphi\varphi)_2)_{11} h_d \\
& + \frac{y_{e10}}{\Lambda^3} e^c ((\ell\chi)_{31} (\varphi\varphi)_{31})_{11} h_d + \frac{y'_\mu}{\Lambda^2} \mu^c (\ell(\varphi\chi)_{32})_{12} h_d + \frac{y_\tau}{\Lambda} \tau^c (\ell\varphi)_{11} h_d + \dots \quad (14)
\end{aligned}$$

In the above superpotential w_ℓ , for each charged lepton, only the lowest order operators in the expansion in powers of $1/\Lambda$ are displayed explicitly. Dots stand for higher dimensional operators. Note that the auxiliary Z_4 symmetry imposes different powers of φ and χ for the electron, mu and tau terms. At LO only the tau mass is generated, the muon and the electron masses are generated by high order contributions. After the flavor symmetry breaking and the electroweak symmetry breaking, the charged leptons acquire masses, and w_ℓ becomes

$$\begin{aligned}
w_\ell = & \left[(y_{e2} - 2y_{e3}) \frac{v_\varphi^3}{\Lambda^3} + (-y_{e4} + 2y_{e5}) \frac{v_\chi^3}{\Lambda^3} + (y_{e7} - 2y_{e8}) \frac{v_\varphi v_\chi^2}{\Lambda^3} + (-y_{e9} + 2y_{e10}) \frac{v_\chi v_\varphi^2}{\Lambda^3} \right] v_d e^c e \\
& + 2y'_\mu \frac{v_\varphi v_\chi}{\Lambda^2} v_d \mu^c \mu + y_\tau \frac{v_\varphi}{\Lambda} v_d \tau^c \tau \\
\equiv & y_e \frac{v_\varphi^3}{\Lambda^3} v_d e^c e + y_\mu \frac{v_\varphi v_\chi}{\Lambda^2} v_d \mu^c \mu + y_\tau \frac{v_\varphi}{\Lambda} v_d \tau^c \tau \quad (15)
\end{aligned}$$

where $v_d = \langle h_d \rangle$, $y_e = y_{e2} - 2y_{e3} + (-y_{e4} + 2y_{e5})\frac{v_\chi^3}{v_\varphi^3} + (y_{e7} - 2y_{e8})\frac{v_\chi^2}{v_\varphi^2} + (-y_{e9} + 2y_{e10})\frac{v_\chi}{v_\varphi}$ and $y_\mu = 2y'_\mu$. As a result, the charged lepton mass matrix is diagonal at LO

$$m_\ell = \begin{pmatrix} y_e \frac{v_\varphi^3}{\Lambda^3} & 0 & 0 \\ 0 & y_\mu \frac{v_\varphi v_\chi}{\Lambda^2} & 0 \\ 0 & 0 & y_\tau \frac{v_\varphi}{\Lambda} \end{pmatrix} v_d \quad (16)$$

It is obvious that the hermitian matrix $m_\ell^\dagger m_\ell$ is invariant under both T and T^2 displayed in the Appendix A, i.e.,

$$T^\dagger m_\ell^\dagger m_\ell T = m_\ell^\dagger m_\ell \quad (17)$$

Conversely, the general matrix invariant under T and T^2 must be diagonal. Consequently the S_4 symmetry is broken to the Z_3 subgroup $H_1^{(1)} \equiv G_\ell$ in the charged lepton sector. The charged lepton masses can be read out directly as

$$m_e = \left| y_e \frac{v_\varphi^3}{\Lambda^3} v_d \right|, \quad m_\mu = \left| y_\mu \frac{v_\varphi v_\chi}{\Lambda^2} v_d \right|, \quad m_\tau = \left| y_\tau \frac{v_\varphi}{\Lambda} v_d \right| \quad (18)$$

we notice that the charged lepton mass hierarchies are naturally generated by the spontaneous symmetry breaking of S_4 symmetry without exploiting the FN mechanism [11]. Using the experimental data on the ratio of the lepton masses, one can estimate the order of magnitude of v_φ/Λ and v_χ/Λ . Assuming that the coefficients y_e , y_μ and y_τ are of $\mathcal{O}(1)$, we obtain

$$\begin{aligned} \frac{m_e}{m_\tau} &\sim \frac{v_\varphi^2}{\Lambda^2} \simeq 3 \times 10^{-4} \\ \frac{m_\mu}{m_\tau} &\sim \frac{v_\chi}{\Lambda} \simeq 6 \times 10^{-2} \end{aligned} \quad (19)$$

Obviously the solution to the above equations is

$$\left(\frac{v_\varphi}{\Lambda}, \frac{v_\chi}{\Lambda} \right) \sim (\pm 1.73 \times 10^{-2}, 6 \times 10^{-2}) \quad (20)$$

we see that the amplitudes of both v_φ/Λ and v_χ/Λ are roughly of the same order about $\mathcal{O}(\lambda_c^2)$, where λ_c is the Cabibbo angle.

4.2 Neutrinos

The superpotential contributing to the neutrino mass is as follows

$$w_\nu = \frac{y_{\nu 1}}{\Lambda} ((\nu^c \ell)_2 \eta)_{11} h_u + \frac{y_{\nu 2}}{\Lambda} ((\nu^c \ell)_{31} \phi)_{11} h_u + \frac{1}{2} M (\nu^c \nu^c)_{11} + \dots \quad (21)$$

where dots denote the higher order contributions, M is a constant with dimension of mass, and the factor $\frac{1}{2}$ is a normalization factor for convenience. The first two terms in Eq.(21) determine the neutrino Dirac mass matrix, and the third term is Majorana mass term. After electroweak and S_4 symmetry breaking, we obtain the following LO contributions to the neutrino Dirac and Majorana mass matrices

$$m_\nu^D = \begin{pmatrix} 2b & a-b & a-b \\ a-b & a+2b & -b \\ a-b & -b & a+2b \end{pmatrix} v_u, \quad M_N = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix} \quad (22)$$

where $v_u = \langle h_u \rangle$, $a = y_{\nu 1} \frac{v_\eta}{\Lambda}$ and $b = y_{\nu 2} \frac{v_\phi}{\Lambda}$. We notice that the Dirac mass matrix is symmetric and it is controlled by two parameters a and b . The eigenvalues of the Majorana matrix M_N are given by

$$M_1 = M, \quad M_2 = M, \quad M_3 = -M \quad (23)$$

The right handed neutrino masses are exactly degenerate, this is a remarkable feature of our model. Integrating out the heavy degrees of freedom, we get the light neutrino mass matrix, which is given by the famous See-Saw relation

$$m_\nu = -(m_\nu^D)^T M_N^{-1} m_\nu^D = -\frac{v_u^2}{M} \begin{pmatrix} 2a^2 + 6b^2 - 4ab & a^2 - 3b^2 + 2ab & a^2 - 3b^2 + 2ab \\ a^2 - 3b^2 + 2ab & a^2 - 3b^2 - 4ab & 2a^2 + 6b^2 + 2ab \\ a^2 - 3b^2 + 2ab & 2a^2 + 6b^2 + 2ab & a^2 - 3b^2 - 4ab \end{pmatrix} \quad (24)$$

The above light neutrino mass matrix m_ν is $2 \leftrightarrow 3$ invariant and it satisfies the magic symmetry $(m_\nu)_{11} + (m_\nu)_{13} = (m_\nu)_{22} + (m_\nu)_{23}$. Therefore it is exactly diagonalized by the TB mixing

$$U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3) \quad (25)$$

The unitary matrix U_ν is written as

$$U_\nu = U_{TB} \text{diag}(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2}) \quad (26)$$

The phases α_1 , α_2 and α_3 are given by

$$\begin{aligned} \alpha_1 &= \arg(-(a - 3b)^2/M) \\ \alpha_2 &= \arg(-4a^2/M) \\ \alpha_3 &= \arg((a + 3b)^2/M) \end{aligned} \quad (27)$$

m_1 , m_2 and m_3 in Eq.(25) are the light neutrino masses,

$$\begin{aligned} m_1 &= |(a - 3b)^2| \frac{v_u^2}{|M|} \\ m_2 &= 4|a^2| \frac{v_u^2}{|M|} \\ m_3 &= |(a + 3b)^2| \frac{v_u^2}{|M|} \end{aligned} \quad (28)$$

Concerning the neutrinos, the S_4 symmetry is spontaneously broken by the VEVs of η and ϕ at the LO. since both $\langle \eta \rangle$ and $\langle \phi \rangle$ are invariant under the actions of $TSTS^2$, TST and S^2 , the flavor symmetry S_4 is broken down to the Klein four subgroup $G_\nu \equiv H_4^{(5)} = \{1, TSTS^2, TST, S^2\}$ in the neutrino sector. We can straightforwardly check that the light neutrino mass matrix m_ν is really invariant under $TSTS^2$, TST and S^2 . On the contrary, the most general neutrino mass matrix invariant under the Klein four group G_ν is given by

$$m_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{11} + m_{12} - m_{22} \\ m_{12} & m_{11} + m_{12} - m_{22} & m_{22} \end{pmatrix} \quad (29)$$

where m_{11} , m_{12} and m_{22} are arbitrary parameters. In the present model, the light neutrino mass matrix is given by Eq.(24), which is a particular version of the neutrino mass matrix in Eq.(29). Since only two parameters a and b are involved in our model, additional constraint has to be satisfied, i.e. $3m_{11}^2 + 4m_{12}m_{11} - 4m_{22}m_{11} - 8m_{12}^2 - 4m_{22}^2 = 0$, which is generally not implied by the invariance under G_ν . This is because that in our model the fields which break S_4 are a doublet η and a triplet ϕ , there are no further flavons transforming as 1_1 or 3_2 which couple to the neutrino sector.

In short summary, at the LO the S_4 flavor symmetry is broken down to Z_3 and Klein four subgroup in the charged lepton and neutrino sector respectively. We have obtained a diagonal and hierarchical charged lepton mass matrix, the heavy neutrino masses are degenerate, and the neutrino mixing matrix is exactly the TB matrix.

4.3 Effective operators

In the previous section, the neutrinos acquire masses via the See-Saw mechanism. It is interesting to note that higher dimension Weinberg operator cloud also contribute to the neutrino mass directly, which may correspond to exchanging some heavy particles rather than the right handed neutrinos ν^c . In the present model, these effective light neutrino mass operators are

$$w_\nu^{eff} = \frac{x}{\Lambda^3}(\ell h_u \ell h_u)_{1_1} \Delta^2 + \frac{y_1}{\Lambda^3}(\ell h_u \ell h_u)_{1_1} (\eta^2)_{1_1} + \frac{y_2}{\Lambda^3}((\ell h_u \ell h_u)_2 (\eta^2)_2)_{1_1} + \frac{z_1}{\Lambda^3}(\ell h_u \ell h_u)_{1_1} (\phi^2)_{1_1} \\ + \frac{z_2}{\Lambda^3}((\ell h_u \ell h_u)_2 (\phi^2)_2)_{1_1} + \frac{z_3}{\Lambda^3}((\ell h_u \ell h_u)_{3_1} (\phi^2)_{3_1})_{1_1} + \frac{w}{\Lambda^3}((\ell h_u \ell h_u)_{3_1} (\eta \phi)_{3_1})_{1_1} \quad (30)$$

With the vacuum configurations displayed in Eq.(12), the high dimension operators w_ν^{eff} leads to the following effective light neutrino mass matrix

$$m_\nu^{eff} = \begin{pmatrix} \alpha + 2\gamma & \beta - \gamma & \beta - \gamma \\ \beta - \gamma & \beta + 2\gamma & \alpha - \gamma \\ \beta - \gamma & \alpha - \gamma & \beta + 2\gamma \end{pmatrix} \frac{v_u^2}{\Lambda} \quad (31)$$

where

$$\alpha = 2x \frac{v_\Delta^2}{\Lambda^2} + 4y_1 \frac{v_\eta^2}{\Lambda^2} + 6z_1 \frac{v_\phi^2}{\Lambda^2} \\ \beta = 2y_2 \frac{v_\eta^2}{\Lambda^2} + 6z_2 \frac{v_\phi^2}{\Lambda^2} \\ \gamma = 4w \frac{v_\eta v_\phi}{\Lambda^2} \quad (32)$$

Obviously m_ν^{eff} have the same texture as that in Eq.(29), and it is remarkable that this mass matrix is diagonalized by TB matrix,

$$U_{TB}^T m_\nu^{eff} U_{TB} = \text{diag}(m_1^{eff}, m_2^{eff}, m_3^{eff}) \quad (33)$$

where m_1^{eff} , m_2^{eff} and m_3^{eff} are the effective light neutrino masses coming from the above high dimension Weinberg operators, they are given by

$$m_1^{eff} = (\alpha - \beta + 3\gamma) \frac{v_u^2}{\Lambda}$$

$$\begin{aligned}
m_1^{eff} &= (\alpha + 2\beta) \frac{v_u^2}{\Lambda} \\
m_3^{eff} &= (-\alpha + \beta + 3\gamma) \frac{v_u^2}{\Lambda}
\end{aligned} \tag{34}$$

If we consider the parameters $y_{\nu 1, \nu 2} \sim \mathcal{O}(1)$, $y_{1,2} \sim \mathcal{O}(1)$, $z_{1,2,3} \sim \mathcal{O}(1)$ and $x \sim w \sim \mathcal{O}(1)$, we get the ratio

$$\frac{m_i^{eff}}{m_i} \sim \frac{M}{\Lambda} \tag{35}$$

We see that the importance of Weinberg operators depends on the relative size of M and Λ . Since we have assumed that the light neutrino masses mainly come from the See-Saw mechanism, the right handed neutrino mass M should be much smaller than the cutoff scale Λ . In the context of a grand unified theory, this corresponds to the requirement that M is of order $\mathcal{O}(M_{GUT})$ rather than of $\mathcal{O}(M_{Planck})$. In some flavor models, right handed neutrino masses are required to be below the cutoff Λ as well, in order to reproduce the experimental value of the small parameter $\Delta m_{sol}^2 / \Delta m_{atm}^2$ [14, 32]. Another convenient way of suppressing the contributions of the effective operators is to introduce auxiliary symmetry further, so that the Weinberg operators arise at much higher order and its contributions can be neglected.

4.4 Extension to the quark sector

The Yukawa superpotentials in the quark sector are

$$w_q = w_u + w_d \tag{36}$$

In the up quark sector, we have

$$\begin{aligned}
w_u &= y_t t^c Q_3 h_u + \sum_{i=1}^3 \frac{y_{ti}}{\Lambda^2} t^c (Q_L \mathcal{O}_i^{(1)})_{1_1} h_u + \sum_{i=1}^2 \frac{y'_{ti}}{\Lambda^3} t^c (Q_L \mathcal{O}_i^{(2)})_{1_1} h_u + \sum_{i=1}^3 \frac{y_{ci}}{\Lambda^2} c^c (Q_L \mathcal{O}_i^{(1)})_{1_2} h_u \\
&+ \sum_{i=1}^2 \frac{y'_{ci}}{\Lambda^3} c^c (Q_L \mathcal{O}_i^{(2)})_{1_2} h_u + \frac{y_{ct}}{\Lambda} c^c Q_3 \theta h_u + \sum_{i=1}^8 \frac{y_{ui}}{\Lambda^4} u^c (Q_L \mathcal{O}_i^{(3)})_{1_1} h_u \\
&+ \sum_{i=1}^2 \frac{y'_{ui}}{\Lambda^3} u^c Q_3 (\mathcal{O}^{(4)})_{1_1} h_u + \dots
\end{aligned} \tag{37}$$

where

$$\begin{aligned}
\mathcal{O}^{(1)} &= \{\varphi\varphi, \varphi\chi, \chi\chi\} \\
\mathcal{O}^{(2)} &= \{\eta^2\Delta, \phi^2\Delta\} \\
\mathcal{O}^{(3)} &= \{\varphi\phi^3, \chi\phi^3, \varphi\eta\phi^2, \chi\eta\phi^2, \varphi\eta^2\phi, \chi\eta^2\phi, \varphi\phi\Delta^2, \chi\phi\Delta^2\} \\
\mathcal{O}^{(4)} &= \{\varphi^3, \varphi\chi^2\}
\end{aligned} \tag{38}$$

The superpotentials contributing to the down quark masses are as follows

$$w_d = \frac{y_b}{\Lambda} b^c Q_3 \theta h_d + \sum_{i=1}^3 \frac{y_{bi}}{\Lambda^2} b^c (Q_L \mathcal{O}_i^{(1)})_{1_2} h_d + \sum_{i=1}^2 \frac{y'_{bi}}{\Lambda^3} b^c (Q_L \mathcal{O}_i^{(2)})_{1_2} h_d + \sum_{i=1}^3 \frac{y_{si}}{\Lambda^2} s^c (Q_L \mathcal{O}_i^{(1)})_{1_2} h_d$$

$$\begin{aligned}
& + \sum_{i=1}^2 \frac{y'_{si}}{\Lambda^3} s^c (Q_L \mathcal{O}_i^{(2)})_{1_2} h_d + \sum_{i=1}^3 \frac{y''_{si}}{\Lambda^3} s^c Q_3 (\mathcal{O}_i^{(5)})_{1_2} h_d + \sum_{i=1}^6 \frac{y_{di}}{\Lambda^3} d^c (Q_L \mathcal{O}_i^{(6)})_{1_1} h_d \\
& + \frac{y'_{d1}}{\Lambda^3} d^c Q_3 (\varphi \chi)_{1_2} \Delta h_d + \sum_{i=1}^9 \frac{y''_{di}}{\Lambda^4} d^c Q_3 (\mathcal{O}_i^{(7)})_{1_1} h_d + \dots
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
\mathcal{O}^{(5)} &= \{\eta^3, \eta\phi^2, \theta^3\} \\
\mathcal{O}^{(6)} &= \{\varphi^2\eta, \varphi^2\phi, \chi^2\eta, \chi^2\phi, \varphi\chi\eta, \varphi\chi\phi\} \\
\mathcal{O}^{(7)} &= \{\varphi^2\theta\Delta, \chi^2\theta\Delta, \eta^4, \eta^2\Delta^2, \eta^2\phi^2, \eta\phi^3, \phi^4, \phi^2\Delta^2, \Delta^4\}
\end{aligned} \tag{40}$$

Since the quantum numbers of b^c and s^c are exactly the same, as is obvious from Table 3, there are no fundamental distinctions between b^c and s^c , we have defined b^c as the one which couples to $Q_3\theta h_d$ in the superpotential w_d . We notice that both the supermultiplets φ , χ and η , ϕ , which control the flavor symmetry breaking in the charged lepton and neutrino sectors respectively, couple to the quarks. Consequently the S_4 flavor symmetry is completely broken in the quark sector. By recalling the vacuum configuration in Eq.(9) and Eq.(12), we can write down the mass matrices for the up and down quarks

$$m_u = \begin{pmatrix} y_{11}^{(u)} \frac{v_\varphi v_\phi^3}{\Lambda^4} & y_{12}^{(u)} \frac{v_\varphi v_\phi^3}{\Lambda^4} & y_{13}^{(u)} \frac{v_\varphi^3}{\Lambda^3} \\ y_{21}^{(u)} \frac{v_\phi^2 v_\Delta}{\Lambda^3} & y_{22}^{(u)} \frac{v_\varphi^2}{\Lambda^2} & y_{23}^{(u)} \frac{v_\theta}{\Lambda} \\ y_{31}^{(u)} \frac{v_\phi^2 v_\Delta}{\Lambda^3} & y_{32}^{(u)} \frac{v_\varphi^2}{\Lambda^2} & y_{33}^{(u)} \end{pmatrix} v_u \tag{41}$$

$$m_d = \begin{pmatrix} y_{11}^{(d)} \frac{v_\varphi^2 v_\phi}{\Lambda^3} & y_{12}^{(d)} \frac{v_\varphi^2 v_\phi}{\Lambda^3} & y_{13}^{(d)} \frac{v_\phi^4}{\Lambda^4} \\ y_{21}^{(d)} \frac{v_\phi^2 v_\Delta}{\Lambda^3} & y_{22}^{(d)} \frac{v_\varphi^2}{\Lambda^2} & y_{23}^{(d)} \frac{v_\theta^3}{\Lambda^3} \\ y_{31}^{(d)} \frac{v_\phi^2 v_\Delta}{\Lambda^3} & y_{32}^{(d)} \frac{v_\varphi^2}{\Lambda^2} & y_{33}^{(d)} \frac{v_\theta}{\Lambda} \end{pmatrix} v_d \tag{42}$$

where $y_{ij}^{(u)}$ and $y_{ij}^{(d)}$ ($i, j = 1, 2, 3$) are the sum of all the different terms appearing in the superpotential, all of them are expected to be of order one. We note that the contribution of $d^c Q_3 (\varphi \chi)_{1_2} \Delta h_d$ vanishes with the LO vacuum alignment, accordingly the (13) element of the down quark mass matrix m_d arise at order $1/\Lambda^4$. Diagonalizing the above quark mass matrices in Eq.(41) and Eq.(42) with the standard perturbation technique, we obtain the quark masses as follows

$$\begin{aligned}
m_u &\simeq \left| y_{11}^{(u)} \frac{v_\varphi v_\phi^3}{\Lambda^4} v_u \right| \\
m_c &\simeq \left| y_{22}^{(u)} \frac{v_\varphi^2}{\Lambda^2} v_u \right| \\
m_t &\simeq \left| y_{33}^{(u)} v_u \right| \\
m_d &\simeq \left| y_{11}^{(d)} \frac{v_\varphi^2 v_\phi}{\Lambda^3} v_d \right| \\
m_s &\simeq \left| y_{22}^{(d)} \frac{v_\varphi^2}{\Lambda^2} v_d \right| \\
m_b &\simeq \left| y_{33}^{(d)} \frac{v_\theta}{\Lambda} v_d \right|
\end{aligned} \tag{43}$$

We see that the quark mass hierarchies are correctly produced if the VEVs v_η , v_ϕ , v_θ and v_Δ are of order $\mathcal{O}(\lambda_c^2 \Lambda)$ as well. This is consistent with our naive expectation that all the VEVs should be of the same order of magnitude. Note that the quark mass hierarchies are generated through the spontaneous breaking of the flavor symmetry instead of the FN mechanism. It is obvious that the mass hierarchies between top and bottom quark mainly come from the symmetry breaking parameter v_θ/Λ , and $\tan\beta \equiv \frac{v_u}{v_d}$ should be of order one in our model. Comparing with the tau lepton mass m_τ predicted in Eq.(18), we see that m_τ and m_b are of the same order, this is consistent with the $b-\tau$ unification predicted in many grand unification models.

For the quark mixing, the CKM matrix elements are estimated as

$$\begin{aligned}
V_{ud} &\simeq V_{cs} \simeq V_{tb} \simeq 1 \\
V_{us}^* &\simeq -V_{cd} \simeq \left(\frac{y_{21}^{(d)}}{y_{22}^{(d)}} - \frac{y_{21}^{(u)}}{y_{22}^{(u)}} \right) \frac{v_\phi^3}{\Lambda v_\varphi^2} \\
V_{ub}^* &\simeq \frac{y_{22}^{(u)} y_{31}^{(d)} - y_{21}^{(u)} y_{32}^{(d)}}{y_{22}^{(u)} y_{33}^{(d)}} \frac{v_\phi^3}{\Lambda^2 v_\theta} \\
V_{cb}^* &\simeq -V_{ts} \simeq \frac{y_{32}^{(d)}}{y_{33}^{(d)}} \frac{v_\varphi^2}{\Lambda v_\theta} \\
V_{td} &\simeq \frac{y_{21}^{(d)} y_{32}^{(d)} - y_{22}^{(d)} y_{31}^{(d)}}{y_{22}^{(d)} y_{33}^{(d)}} \frac{v_\phi^3}{\Lambda^2 v_\theta}
\end{aligned} \tag{44}$$

We see that the correct orders of the CKM matrix elements are reproduced with the exception of Cabibbo angle. The V_{us} (or V_{cd}) is the combination of two independent contributions of order λ_c^2 , we need an accidental enhancement of the combination $\left(\frac{y_{21}^{(d)}}{y_{22}^{(d)}} - \frac{y_{21}^{(u)}}{y_{22}^{(u)}} \right)$ of order $1/\lambda_c$ in order to obtain the correct Cabibbo angle.

5 Phenomenological implications

In the following we shall study the constraints on the model imposed by the observed values of $\Delta m_{sol}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{atm}^2 \equiv |m_3^2 - m_1^2(m_2^2)|$. The important physical consequences of our model are investigated in details, and the corresponding predictions are presented. In this section we mainly concentrate on the neutrino sector. We assume that the right handed neutrino mass M is much smaller than the cutoff scale Λ of the theory, then the light neutrino masses are dominantly generated via the See-Saw mechanism.

5.1 The neutrino mass spectrum

According to Eq.(28), the light neutrino mass spectrum is controlled by two parameters a and b , which are in general both complex numbers. For convenience, we define

$$\frac{b}{a} = R e^{i\Phi} \tag{45}$$

with $R = |\frac{b}{a}|$. As we will see in the following, all the low energy observables can be expressed in terms of only three independent quantities: the ratio R , the relative phase Φ between a

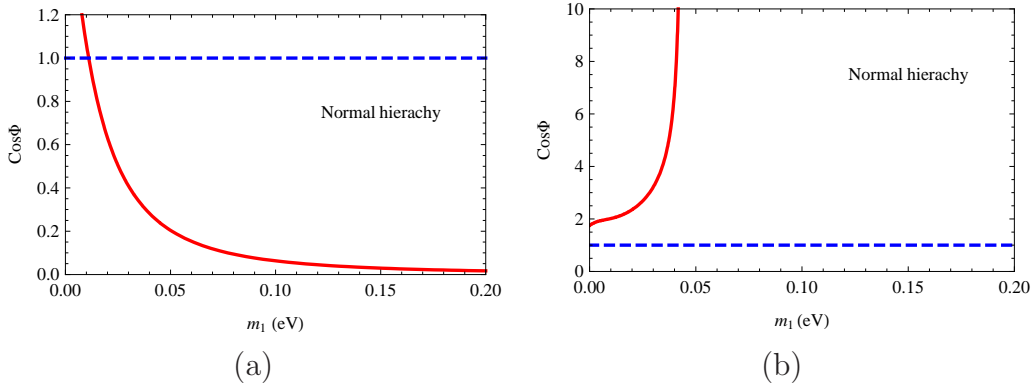


Figure 1: The variation of $\cos \Phi$ with respect to the lightest neutrino mass m_1 for the normal hierarchy spectrum. In Fig. 1a, $\cos \Phi$ is taken to be the expression in Eq.(48), and Fig. 1b corresponds to the value of $\cos \Phi$ in Eq.(49).

and b , and the lightest neutrino mass. Experimentally, only two spectrum observables Δm_{sol}^2 and Δm_{atm}^2 have been measured, therefore the light neutrino mass spectrum can be normal hierarchy(NH) or inverted hierarchy(IH). The ratio between Δm_{sol}^2 and Δm_{atm}^2 is given by

$$\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} = \frac{15 - 81R^4 - 18R^2 - 36R^2 \cos^2 \Phi + 12R(1 + 9R^2) \cos \Phi}{24R(1 + 9R^2) |\cos \Phi|} \quad (46)$$

where we have taken $\Delta m_{atm}^2 = |m_3^2 - m_1^2|$ for both the NH and IH neutrino spectrums for convenience. Moreover, we have the following relationships for the neutrino masses

$$\begin{aligned} \frac{16m_1^2}{m_2^2} &= 1 + 81R^4 + 18R^2 + 36R^2 \cos^2 \Phi - 12R(1 + 9R^2) \cos \Phi \\ \frac{16m_3^2}{m_2^2} &= 1 + 81R^4 + 18R^2 + 36R^2 \cos^2 \Phi + 12R(1 + 9R^2) \cos \Phi \end{aligned} \quad (47)$$

Then the parameters R and $\cos \Phi$ can be expressed in terms of light neutrino mass as follows

$$\begin{cases} R = \frac{1}{3} \sqrt{\frac{2(m_3 + m_1)}{m_2} - 1} \\ \cos \Phi = \frac{m_3 - m_1}{m_2} \frac{1}{\sqrt{\frac{2(m_3 + m_1)}{m_2} - 1}} \end{cases} \quad (48)$$

or

$$\begin{cases} R = \frac{1}{3} \sqrt{\frac{2|m_3 - m_1|}{m_2} - 1} \\ \cos \Phi = \frac{m_3^2 - m_1^2}{m_2 |m_3 - m_1|} \frac{1}{\sqrt{\frac{2|m_3 - m_1|}{m_2} - 1}} \end{cases} \quad (49)$$

These results hold for both the normal hierarchy and inverted hierarchy spectrum. In the case of normal hierarchy, m_2 and m_3 can be expressed as functions of the lightest neutrino mass: $m_2 = \sqrt{m_1^2 + \Delta m_{sol}^2}$ and $m_3 = \sqrt{m_1^2 + \Delta m_{atm}^2}$. For the inverted hierarchy, m_3 is the lightest neutrino mass, the remaining two masses are $m_1 = \sqrt{m_3^2 + \Delta m_{atm}^2}$ and $m_2 = \sqrt{m_3^2 + \Delta m_{sol}^2 + \Delta m_{atm}^2}$. As a result, taking into account the experimental information on

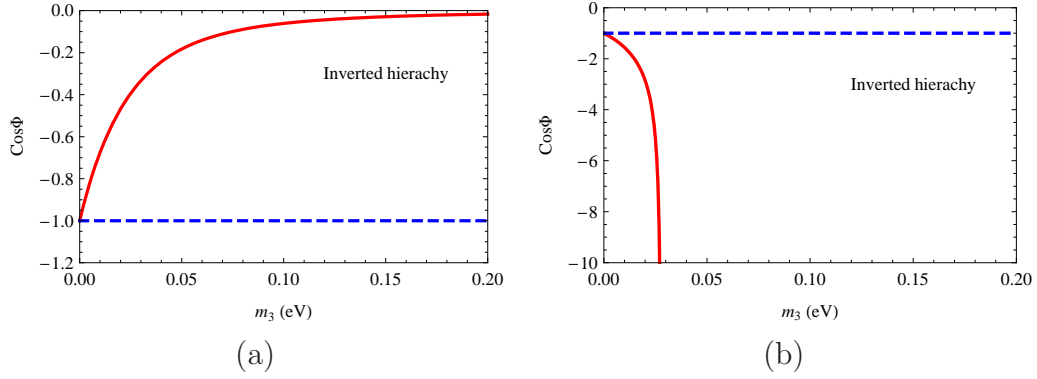


Figure 2: $\cos \Phi$ as a function of the lightest neutrino mass m_3 for the inverted hierarchy spectrum. Fig. 2a and Fig. 2b are for the $\cos \Phi$ values in Eq.(48) and Eq.(49) respectively.

Δm_{sol}^2 and Δm_{atm}^2 , there is only one real parameter undetermined, and it is chose to be the lightest neutrino mass $m_l(m_1$ or $m_3)$ in the present work. Hence our model is quite predictive. We display $\cos \Phi$ as a function of the lightest neutrino mass in Fig.1 and Fig.2 for the normal hierarchy and inverted hierarchy respectively, where the best fit values of $\Delta m_{sol}^2 = 7.65 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{atm}^2 = 2.40 \times 10^{-3} \text{ eV}^2$ have been used. For the solution of R and $\cos \Phi$ shown in Eq.(49), we can clearly see that the corresponding value of $|\cos \Phi|$ would be larger than 1 in the case of both normal hierarchy and inverted hierarchy spectrum. Furthermore, we have verified that $|\cos \Phi|$ is always larger than 1 for the 3σ range of Δm_{sol}^2 and Δm_{atm}^2 , therefore the solution in Eq.(49) can be disregarded thereafter. From the condition $|\cos \Phi| \leq 1$, we obtain the following constraints on the lightest neutrino mass,

$$\begin{aligned} m_1 &\geq 0.011 \text{ eV}, & \text{Normal hierarchy} \\ m_3 &> 0.0 \text{ eV}, & \text{Inverted hierarchy} \end{aligned} \quad (50)$$

For the NH spectrum, we have a lower bound on m_1 , which is satisfied for $\Phi = 0$. The corresponding values of $\cos \Phi$ are positive, then Φ is in the range of $0 \sim \pi/2$ or $3\pi/2 \sim 2\pi$. In the case of IH, $\cos \Phi$ is negative so that Φ varies between $\pi/2$ to $3\pi/2$. From Fig. 2 we can see that $\cos \Phi$ is very close to -1 for m_3 tending to zero, the lightest neutrino mass m_3 is less constrained.

5.2 Neutrinoless double beta decay

Neutrinoless double beta decay ($0\nu 2\beta$) is a sensitive probe to the scale of the neutrino masses, it is a very slow lepton-number-violating nuclear transition that occurs if neutrinos have mass and are Majorana particles. The rate of $0\nu 2\beta$ decay is determined by the nuclear matrix elements and the effective $0\nu 2\beta$ -decay mass $|m_{\beta\beta}|$, which is defined as $|m_{\beta\beta}| = |\sum_k (U_{PMNS})_{ek}^2 m_k|$. In the present model it is given by

$$\begin{aligned} |m_{\beta\beta}| &= |2a^2 + 6b^2 - 4ab| \frac{v_u^2}{|M|} \\ &= \frac{m_2}{2} \left[1 + 9R^4 + 4R^2 + 6R^2 \cos(2\Phi) - 12R^3 \cos \Phi - 4R \cos \Phi \right]^{1/2} \end{aligned} \quad (51)$$

By using Eq.(48), we can express $|m_{\beta\beta}|$ in terms of the lightest neutrino mass m_l , the corresponding results are shown in Fig.3. The vertical line represents the future sensitivity of the KATRIN experiment [41], the horizontal ones denote the present bound from the Heidelberg-Moscow experiment [42] and the future sensitivity of some $0\nu 2\beta$ decay experiments, which are 15 meV, 20 meV and 90 meV, respectively of CUORE [43], Majorana [44]/GERDA III [45] and GERDA II experiments. From Fig. 3 we conclude that for the allowed values of m_l , the predictions for $m_{\beta\beta}$ approach the future experimental sensitivity. For the NH spectrum, the effective mass $m_{\beta\beta}$ can reach a very low value about 7.8 meV. Whereas the lower bound for $m_{\beta\beta}$ is approximately 44.3 meV in the case of IH. A combined measurement of the effective mass $m_{\beta\beta}$ and the lightest neutrino mass can determine whether the neutrino spectrum is NH or IH in our model.

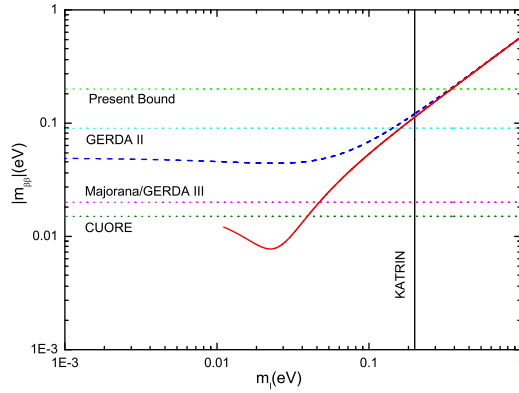


Figure 3: $m_{\beta\beta}$ as a function of the lightest neutrino mass m_l , the solid and dashed lines represent the NH and IH cases respectively.

5.3 Beta decay

One can directly search for the kinetic effect of nonzero neutrino masses in beta decay by modification of the Kurie plot. This search is sensitive to neutrino masses regardless of whether the neutrinos are Dirac or Majorana particles. For small neutrino masses, this effect will occur near to the end point of the electron energy spectrum and will be sensitive to the quantity $m_\beta = [\sum_k |(U_{PMNS})_{ek}|^2 m_k^2]^{1/2}$. For the present model, we have

$$m_\beta = \frac{1}{\sqrt{3}}(2m_1^2 + m_2^2)^{1/2} \quad (52)$$

This result holds for both the NH and IH spectrums. In Fig.4 we plot m_β versus the lightest neutrino mass m_l , the horizontal line represents the future sensitivity of 0.2 eV from the KATRIN experiment.

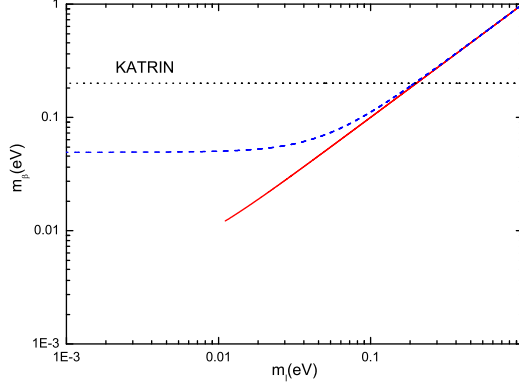


Figure 4: Variation of m_β with respect to the lightest neutrino mass m_l , the solid and dashed lines represent the NH and IH spectrum respectively.

5.4 Sum of the neutrino masses

The sum of the neutrino masses $\sum_k m_k$ is constrained by the cosmological observation. In Fig. 5 we display the sum of the neutrino masses as a function of the lightest neutrino mass m_l . The vertical line denotes the future sensitivity of KATRIN experiment, and the horizontal lines are the cosmological bounds [46]. There are typically five representative combinations of the cosmological data, which lead to increasingly stronger upper bounds on the sum of the neutrino masses. We show the two strongest ones in Fig. 5. The first one at 0.60 eV corresponds to the combination of the Cosmic Microwave Background (CMB) anisotropy data (from WMAP 5y [47], Arcminute Cosmology Bolometer Array Receiver (ACBAR) [48], Very Small Array (VSA) [49], Cosmic Background Imager (CBI) [50] and BOOMERANG [51] experiments) plus the large-scale structure (LSS) information on galaxy clustering (from the Luminous Red Galaxies Sloan Digital Sky Survey (SDSS) [52]) plus the Hubble Space Telescope (HST) plus the luminosity distance SN-Ia data of [53] and finally plus the BAO data from [54]. The second one at 0.19 eV corresponds to all the previous data combined to the small scale primordial spectrum from Lyman-alpha ($\text{Ly}\alpha$) forest clouds [55]. We see that the current cosmological information on the sum of the neutrino masses can hardly distinguish the NH spectrum from the IH spectrum.

5.5 The Majorana CP violating phases

In the standard parametrization [5], the lepton PMNS mixing matrix is defined by

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) \quad (53)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with $\theta_{ij} \in [0, \pi/2]$, δ is the Dirac CP violating phase, α_{21} and α_{31} are the two Majorana CP violating phases, all the three CP violating phases δ , α_{21} and

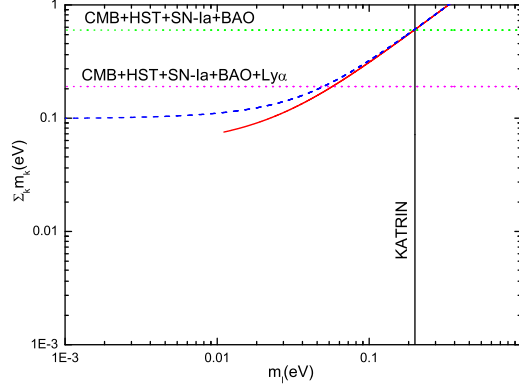


Figure 5: The sum of the neutrino masses $\sum_k m_k$ versus the lightest neutrino mass m_l , the solid and dashed lines represent the NH and IH spectrum respectively.

α_{31} are allowed to vary in the range of $0 \sim 2\pi$. Recalling that the leptonic mixing matrix is given by Eq.(26) at LO, in the standard parametrization it is,

$$U_{PMNS} = e^{-i\alpha_1/2} \text{diag}(1, 1, -1) U_{TB} \text{diag}(1, e^{i(\alpha_1-\alpha_2)/2}, e^{i(\alpha_1-\alpha_3)/2}) \quad (54)$$

where the overall phase $e^{-i\alpha_1/2}$ can be absorbed into the charged lepton fields. Comparing Eq.(54) with Eq.(53), we can identify the two CP violating phases as

$$\alpha_{21} = \alpha_1 - \alpha_2, \quad \alpha_{31} = \alpha_1 - \alpha_3 \quad (55)$$

Similar to other low energy observables, α_{21} and α_{31} can be written as functions of the parameters R and Φ ,

$$\begin{cases} \sin \alpha_{21} = \frac{9R^2 \sin(2\Phi) - 6R \sin \Phi}{1 + 9R^2 - 6R \cos \Phi}, \\ \cos \alpha_{21} = \frac{1 + 9R^2 \cos(2\Phi) - 6R \cos \Phi}{1 + 9R^2 - 6R \cos \Phi} \end{cases} \quad (56)$$

$$\begin{cases} \sin \alpha_{31} = \frac{12R(1-9R^2) \sin \Phi}{1 + 81R^4 + 18R^2 - 36R^2 \cos^2 \Phi} \\ \cos \alpha_{31} = -\frac{1 + 81R^4 - 36R^2 + 18R^2 \cos(2\Phi)}{1 + 81R^4 + 18R^2 - 36R^2 \cos^2 \Phi} \end{cases} \quad (57)$$

Note that the relations between R , Φ and the light neutrino masses are displayed in Eq.(48). In contrast to other low energy observables such as the light neutrino masses, $m_{\beta\beta}$ and m_β etc., the Majorana phases α_{21} and α_{31} depend on both $\cos \Phi$ and $\sin \Phi$, and not only on $\cos \Phi$. In Fig.6 we show the behavior of the Majorana phases α_{21} and α_{31} with respect to the lightest neutrino mass m_l , where we choose $\sin \Phi > 0$ for illustration.

It is well-known that the See-Saw mechanism provides an elegant explanation for the smallness of the neutrino mass, meanwhile the baryon asymmetry may be produced through the out of equilibrium CP violating decays of the right handed neutrino ν^c . As is shown in Eq.(23), at LO the heavy neutrino masses are exactly degenerate in our model, then

leptogenesis can be naturally implemented via the so-called resonant leptogenesis mechanism [56]. Since more other subtle issues are involved in the resonant leptogenesis, the analysis of whether the observed baryon asymmetry can be naturally generated in our model is beyond the range of the present paper, which will be discussed in future work [57].

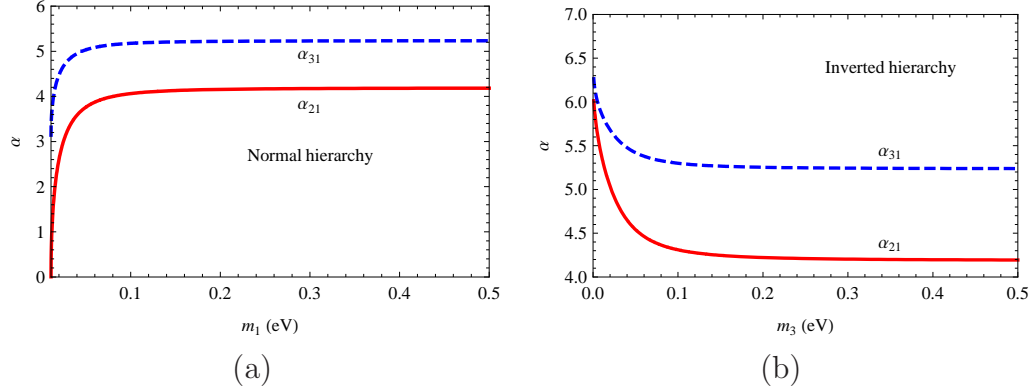


Figure 6: The dependence of the Majorana CP violating phases α_{21} and α_{31} on the lightest neutrino mass m_l . Solid and dashed lines refer to α_{21} and α_{31} respectively. Fig.6a corresponds to the NH mass spectrum, and Fig.6b is for the IH case, where $\sin \Phi$ is taken to be positive.

The phenomenological consequences for the LO order predictions of our model have been analyzed. As we shall show in the next section that our model gets corrections when higher dimensional operators are included in the Lagrangian. These corrections modify the leading order predictions by terms of relative order λ_c^2 , hence the results presented so far are still correct approximately. However, we would like to note that due to the next to the leading order contributions, a cancellation could be present for the effective $0\nu 2\beta$ -decay mass, consequently $m_{\beta\beta}$ could reach zero in the NH case.

6 Next to the leading order corrections

The results of the previous section hold to first approximation. At the next to leading order (NLO), the superpotentials w_v , w_ℓ , w_ν , w_u and w_d are corrected by higher dimensional operators compatible with the symmetry of the model, whose contributions are suppressed by at least one additional power of Λ . The residual Klein four and Z_3 symmetry in the neutrino and the charged lepton sectors at LO would be broken completely by the NLO contributions. The NLO terms in the driving superpotential leads to small deviation from the LO vacuum alignment. The masses and mixing matrices are corrected by both the shift of the vacuum configuration and the NLO operators in the Yukawa superpotentials w_ℓ , w_ν , w_u and w_d . In the following, the NLO corrections to the vacuum alignment and the mass matrices will be discussed one by one, and the resulting physical effects are studied.

6.1 Corrections to the vacuum alignment

The NLO operators of the driving superpotential w_v and the corresponding corrections to the LO vacuum alignment in Eq.(9) and Eq.(12) are discussed in the Appendix B in details.

The inclusion of the higher dimensional operators results in the shift of the VEVs of the flavon fields, the vacuum configuration is modified into

$$\begin{aligned}\langle\varphi\rangle &= (\delta v_{\varphi_1}, v_\varphi + \delta v_{\varphi_2}, \delta v_{\varphi_3}), & \langle\chi\rangle &= (\delta v_{\chi_1}, v_\chi, \delta v_{\chi_3}), \\ \langle\eta\rangle &= (v_\eta, v_\eta + \delta v_{\eta_2}), & \langle\phi\rangle &= (v_\phi + \delta v_\phi, v_\phi + \delta v_\phi, v_\phi + \delta v_\phi), \\ \langle\theta\rangle &= v_\theta + \delta v_\theta\end{aligned}\tag{58}$$

where v_χ and v_η are still undetermined, and the VEV v_Δ is not corrected by the NLO terms. All the corrections are suppressed by $1/\Lambda$, and the shift of $\langle\phi\rangle$ turns out to be proportional to its LO VEV. Since all the VEVs are required to be of order $\mathcal{O}(\lambda_c^2\Lambda)$, we expect these corrections would modify the LO VEVs by terms of relative order λ_c^2 .

6.2 Corrections to the mass matrices

The corrections to the fermion mass matrices originate from two sources: the first is the higher dimensional operators in the Yukawa superpotentials w_ℓ , w_ν , w_u and w_d , and the second is the deviation from the LO vacuum alignment, which is induced by the NLO terms in the driving potential. As a result, at NLO the mass matrices are the sum of the contributions of higher dimension operators evaluated with the insertion of the the LO VEV, and those from the LO superpotentials evaluated with the NLO VEVs.

For the charged leptons, the superpotential w_ℓ is corrected by the following sixteen NLO operators

$$\begin{aligned}e^c(\ell\varphi^3)_{12}\theta h_d, & \quad e^c(\ell\varphi^2\chi)_{12}\theta h_d, & e^c(\ell\varphi\chi^2)_{12}\theta h_d, & \quad e^c(\ell\chi^3)_{12}\theta h_d, \\ e^c(\ell\varphi\eta^2)_{12}\Delta h_d, & \quad e^c(\ell\chi\eta^2)_{12}\Delta h_d, & e^c(\ell\varphi\eta\phi)_{12}\Delta h_d, & \quad e^c(\ell\chi\eta\phi)_{12}\Delta h_d, \\ e^c(\ell\varphi\phi^2)_{12}\Delta h_d, & \quad e^c(\ell\chi\phi^2)_{12}\Delta h_d, & \mu^c(\ell\varphi^2)_{11}\theta h_d, & \quad \mu^c(\ell\chi^2)_{11}\theta h_d, \\ \mu^c(\ell\varphi\chi)_{11}\theta h_d, & \quad \mu^c(\ell\phi^2)_{11}\Delta h_d, & \mu^c(\ell\eta\phi)_{11}\Delta h_d, & \quad \tau^c(\ell\chi)_{12}\theta h_d\end{aligned}\tag{59}$$

Taking into account the contributions of the modified vacuum alignment at NLO, each diagonal entry of the charged lepton mass matrix receives a small correction factor, while the off-diagonal entries become non-zero and of the order of the diagonal term in each row multiplied by ε , which parameterizes the ratio VEV/Λ with order $\mathcal{O}(\lambda_c^2)$. Then we have

$$m_\ell = \begin{pmatrix} m_{11}^\ell \varepsilon^2 & m_{12}^\ell \varepsilon^3 & m_{13}^\ell \varepsilon^3 \\ m_{21}^\ell \varepsilon^2 & m_{22}^\ell \varepsilon & m_{23}^\ell \varepsilon^2 \\ m_{31}^\ell \varepsilon & m_{32}^\ell \varepsilon & m_{33}^\ell \end{pmatrix} \varepsilon v_d\tag{60}$$

where the coefficients $m_{ij}^\ell (i, j = 1, 2, 3)$ are order one unspecified constants. The hermitian matrix $m_\ell^\dagger m_\ell$ is diagonalized by the unitary matrix U_ℓ , which exactly corresponds to the transformation of the charged leptons used to diagonalize m_ℓ ,

$$U_\ell^\dagger m_\ell^\dagger m_\ell U_\ell \simeq \text{diag}(|m_{11}^\ell \varepsilon^3|^2, |m_{22}^\ell \varepsilon^2|^2, |m_{33}^\ell \varepsilon|^2) v_d^2\tag{61}$$

The charged lepton masses are modified by terms of relative order ε with respect to LO results, consequently the NLO corrections don't spoil the charged lepton mass hierarchies predicted

at LO. The unitary matrix U_ℓ is approximately given by

$$U_\ell \simeq \begin{pmatrix} 1 & (\frac{m_{21}^\ell}{m_{22}^\ell}\varepsilon)^* & (\frac{m_{31}^\ell}{m_{33}^\ell}\varepsilon)^* \\ -\frac{m_{21}^\ell}{m_{22}^\ell}\varepsilon & 1 & (\frac{m_{32}^\ell}{m_{33}^\ell}\varepsilon)^* \\ -\frac{m_{31}^\ell}{m_{33}^\ell}\varepsilon & -\frac{m_{32}^\ell}{m_{33}^\ell}\varepsilon & 1 \end{pmatrix} \quad (62)$$

Then we turn to the neutrino sector. The NLO correction to the Majorana masses of the right handed neutrino arises at order $1/\Lambda$, the corresponding higher dimensional operator is $(\nu^c \nu^c)_{11} \theta^2$, whose contribution can be completely absorbed into the redefinition of the mass parameter M . The NLO corrections to the neutrino Dirac couplings are

$$\frac{y_{\nu 1}}{\Lambda}(\nu^c \ell \delta \eta)_{11} h_u + \frac{y_{\nu 2}}{\Lambda}(\nu^c \ell \delta \phi)_{11} h_u + \frac{x_{\nu 1}}{\Lambda^2}(\nu^c \ell \eta)_{12} \theta h_u + \frac{x_{\nu 2}}{\Lambda^2}(\nu^c \ell \phi)_{12} \theta h_u \quad (63)$$

where $\delta \eta$ and $\delta \phi$ represent the shifted VEVs of the flavons η and ϕ respectively. Through redefining the LO parameters $a \rightarrow a - x_{\nu 1} \frac{v_\eta v_\theta}{\Lambda^2}$ and $b \rightarrow b + y_{\nu 2} \frac{\delta v_\phi}{\Lambda}$, the NLO corrections to m_ν^D are

$$\delta m_\nu^D = \begin{pmatrix} 0 & \delta_2 & \delta_1 - \delta_2 \\ -\delta_2 & \delta_1 & \delta_2 \\ \delta_1 + \delta_2 & -\delta_2 & 0 \end{pmatrix} v_u \quad (64)$$

where $\delta_1 = y_{\nu 1} \frac{\delta v_{\eta 2}}{\Lambda} + 2x_{\nu 1} \frac{v_\eta v_\theta}{\Lambda^2}$ and $\delta_2 = x_{\nu 2} \frac{v_\phi v_\theta}{\Lambda^2}$. We notice that both δ_1 and δ_2 are of order εa (or εb). Therefore, the NLO corrections to the light neutrino mass matrix are given by

$$\begin{aligned} \delta m_\nu &= -(m_\nu^D)^T M_N^{-1} \delta m_\nu^D - (\delta m_\nu^D)^T M_N^{-1} m_\nu^D \\ &= \frac{v_u^2}{M} \begin{pmatrix} -2(a-b)\delta_1 & -(2a+b)\delta_1 - 6b\delta_2 & -b(\delta_1 - 6\delta_2) \\ -(2a+b)\delta_1 - 6b\delta_2 & 2b(\delta_1 + 3\delta_2) & -(2a+b)\delta_1 \\ -b(\delta_1 - 6\delta_2) & -(2a+b)\delta_1 & -2(a-b)\delta_1 - 6b\delta_2 \end{pmatrix} \end{aligned} \quad (65)$$

Diagonalizing the modified light neutrino mass matrix, we obtain the neutrino masses to LO in $\delta_{1,2}$ as follows,

$$\begin{aligned} m_1 &= \left| (a-3b)^2 + (a-3b)\delta_1 \right| \frac{v_u^2}{|M|} \\ m_2 &= 4 \left| a^2 + a\delta_1 \right| \frac{v_u^2}{|M|} \\ m_3 &= \left| (a+3b)^2 + (a+3b)\delta_1 \right| \frac{v_u^2}{|M|} \end{aligned} \quad (66)$$

The PMNS matrix becomes $U_{PMNS} = U_\ell^\dagger U_\nu$, where U_ℓ associated with the diagonalization of the charged lepton mass matrix is given by Eq.(62), and the unitary matrix U_ν diagonalizes the neutrino mass matrix $m_\nu + \delta m_\nu$ including the NLO contributions. The parameters of the lepton mixing matrix are modified as

$$\begin{aligned} |U_{e3}| &= \frac{1}{\sqrt{2}} \left| \frac{1}{6(|a|^2 + 9|b|^2)(ab^* + a^*b)} [(a+3b)^2(a^*\delta_1^* + 6b^*\delta_2^*) - (a^* - 3b^*)^2(a\delta_1 + 6b\delta_2)] \right. \\ &\quad \left. + \left(\frac{m_{21}^\ell}{m_{22}^\ell} \varepsilon \right)^* - \left(\frac{m_{31}^\ell}{m_{33}^\ell} \varepsilon \right)^* \right| \end{aligned}$$

$$\begin{aligned}
\sin^2 \theta_{12} &= \frac{1}{3} \left[1 - \frac{m_{21}^\ell}{m_{22}^\ell} \varepsilon - \frac{m_{31}^\ell}{m_{33}^\ell} \varepsilon - \left(\frac{m_{21}^\ell}{m_{22}^\ell} \varepsilon \right)^* - \left(\frac{m_{31}^\ell}{m_{33}^\ell} \varepsilon \right)^* \right] \\
\sin^2 \theta_{23} &= \frac{1}{2} + \frac{1}{2(|a|^2 + 9|b|^2)(ab^* + a^*b)} \left[ab(a^* \delta_1^* + 6b^* \delta_2^*) + a^* b^* (a \delta_1 + 6b \delta_2) \right] \\
&\quad + \frac{1}{2} \left[\frac{m_{32}^\ell}{m_{33}^\ell} \varepsilon + \left(\frac{m_{32}^\ell}{m_{33}^\ell} \varepsilon \right)^* \right]
\end{aligned} \tag{67}$$

We see that the neutrino masses and mixing angles receive corrections of order λ_c^2 with respect to LO results. The value of $\sin^2 \theta_{12}$ is still within the 3σ range of global data fit, and the corrections to both θ_{23} and θ_{13} are within the current data uncertainties as well. In particular, a non-vanishing θ_{13} of order λ_c^2 is close to the reach of the next generation neutrino oscillation experiments and will provide a valuable test of model.

The NLO corrections to the mass matrices in quark sector have been analyzed following the same method as that for the lepton sector. Since every entry of the mass matrices m_u and m_d in Eq.(41) and Eq.(42) is nonvanishing, the NLO contributions leads to small corrections of relative order λ_c^2 in each entry. Consequently the quark masses and mixing angles are corrected by terms of relative order λ_c^2 with respect to LO results, the successful LO predictions are not spoiled.

7 Conclusion

We have constructed a SUSY model for fermion masses and flavor mixings based on the flavor symmetry $S_4 \times Z_3 \times Z_4$, the neutrino masses are assumed to be generated through the See-Saw mechanism. At LO the S_4 symmetry is broken down to Klein four and Z_3 symmetry in the neutrino and charged lepton sector respectively, this breaking chain exactly leads to the TB mixing. It is remarkable that the mass hierarchies among the charged leptons are controlled by the spontaneous breaking of the flavor symmetry. We further extend the flavor symmetry to the quark sector, where the S_4 symmetry is completely broken. The correct orders of quark masses and CKM matrix elements are generated with the exception of the mixing angle between the first two generations, which requires a small accidental enhancement.

We have carefully analyzed the NLO contributions due to higher dimensional operators which modify both the Yukawa couplings and the LO vacuum alignment, and we have verified that all the fermion masses and mixing angles are corrected by terms of relative order λ_c^2 with respect to the LO results. As a result, the successful LO predictions are not spoiled. Particularly we expect the mixing angle θ_{13} would be of order λ_c^2 , it is within the sensitivity of the experiments which are now in preparation and will take data in the near future [58, 59]. Precise measurement of θ_{13} is an important test to our model.

The phenomenological consequences of our model are analyzed in detail. The low energy observables including the neutrino mass squared difference, neutrinoless double decay, beta decay, the sum of the neutrino masses and the Majorana CP violating phases are considered. All the low energy observables can be expressed in terms of three independent parameters: the ratio $R = |b/a|$, the relative phase Φ between a and b and the lightest neutrino mass m_l . Once the parameters are fixed to match Δm_{sol}^2 and Δm_{atm}^2 , there is only one parameter left, which is chose to be m_l in the present work. Both the normal and inverted hierarchy

neutrino spectrum are allowed in our model. For normal hierarchy there is a low bound on m_1 of approximately 0.011 eV. In the case of inverted hierarchy, m_3 is less constrained and we only obtain the trivial constraint that m_3 should be positive. The lower bounds of the effective mass $m_{\beta\beta}$ are approximately 7.8 meV and 44.3 meV for the NH and IH spectrum respectively. A combined measurement of $m_{\beta\beta}$ and the lightest neutrino mass can distinguish the NH from the IH spectrum. The Majorana CP violating phases depend both on $\cos\Phi$ and $\sin\Phi$, whereas only $\cos\Phi$ is involved in other low energy observables. It is remarkable that the right handed neutrino masses are exactly degenerate at LO, the baryon asymmetry may be generated via the resonant leptogenesis.

Acknowledgements

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Appendix A: Representation matrices of the S_4 group

In this appendix, we explicitly show the representation matrices of the S_4 group for the five irreducible representations. The matrices for the generators S and T depend on the representations as follows

$$\begin{array}{lll}
1_1, & S = 1, & T = 1 \\
1_2, & S = -1, & T = 1 \\
2, & S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} \\
3_1, & S = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, & T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \\
3_2, & S = \frac{-1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, & T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}
\end{array}$$

where $\omega = e^{2\pi i/3} = (-1 + \sqrt{3})/2$. In the identity representation 1_1 , all the elements are mapped onto the number 1. In the antisymmetric representation 1_2 , the group elements correspond to 1 or -1 respectively for even permutation and odd permutation. For the 2 representation, the representation matrices are as follows

$$\begin{aligned}
\mathcal{C}_1 : & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\mathcal{C}_2 : & STS^2 = T^2S = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \quad TSTS^2 = TST = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad ST^2 = S^2TS = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \\
\mathcal{C}_3 : & TS^2T^2 = S^2 = T^2S^2T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\mathcal{C}_4 : & T = S^2T = TS^2 = S^2TS^2 = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad T^2 = S^2T^2 = STS = T^2S^2 = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} \\
\mathcal{C}_5 : & S = S^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T^2ST = TS = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, \quad ST = TST^2 = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}
\end{aligned}$$

For the 3_1 representation, the representation matrices are

$$\begin{aligned}
\mathcal{C}_1 : & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\mathcal{C}_2 : & STS^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \omega \\ 0 & \omega^2 & 0 \end{pmatrix}, \quad TSTS^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad ST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \\
& S^2TS = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \omega^2 \\ 0 & \omega & 0 \end{pmatrix}, \quad TST = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}, \quad T^2S = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & 2 & -\omega \\ 2\omega & -\omega^2 & 2 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_3: TS^2T^2 &= \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}, S^2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, T^2S^2T = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & -1 & 2\omega^2 \\ 2\omega^2 & 2\omega & -1 \end{pmatrix} \\
\mathcal{C}_4: T &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, T^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, T^2S^2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega & -\omega & 2\omega \\ 2\omega^2 & 2\omega^2 & -\omega^2 \end{pmatrix}, \\
S^2T &= \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & -\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \end{pmatrix}, S^2TS^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & -\omega^2 & 2 \\ 2\omega^2 & 2 & -\omega \end{pmatrix}, STS = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & -\omega & 2 \\ 2\omega & 2 & -\omega^2 \end{pmatrix}, \\
S^2T^2 &= \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & -\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \end{pmatrix}, TS^2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & -\omega^2 & 2\omega^2 \\ 2\omega & 2\omega & -\omega \end{pmatrix} \\
\mathcal{C}_5: S &= \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, T^2ST = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & 2\omega^2 & -\omega^2 \\ 2\omega & -\omega & 2\omega \end{pmatrix}, ST = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega & 2\omega & -\omega \\ 2\omega^2 & -\omega^2 & 2\omega^2 \end{pmatrix}, \\
TS &= \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \\ 2 & -\omega & 2\omega^2 \end{pmatrix}, TST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \\ 2 & -\omega^2 & 2\omega \end{pmatrix}, S^3 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & 2\omega & -1 \\ 2\omega & -1 & 2\omega^2 \end{pmatrix}
\end{aligned}$$

Since the signs of the generator S are opposite in 3_1 and 3_2 representations, the representation matrices for the 3_2 representation can be found from those of the 3_1 representation: the matrices are exactly the same for \mathcal{C}_1 , \mathcal{C}_3 and \mathcal{C}_4 classes, whereas they are the opposite for \mathcal{C}_2 and \mathcal{C}_5 .

Appendix B: NLO corrections to the vacuum alignment

In this appendix, we will analyze the NLO corrections to the vacuum alignment induced by the higher dimensional operators. At NLO the driving superpotential dependent on the driving fields is modified into

$$w_v + \Delta w_v \quad (68)$$

where w_v is the LO contributions shown in Eq.(7), which are of dimension three. Δw_v is most general set of terms suppressed by one power of the cutoff, which are linear in the driving fields and are invariant under the symmetry of the model. Concretely Δw_v is given by

$$\Delta w_v = \frac{1}{\Lambda} \left[\sum_{i=1}^2 p_i \mathcal{O}_i^\varphi + \sum_{i=1}^5 x_i \mathcal{O}_i^{\xi'} + \sum_{i=1}^3 t_i \mathcal{O}_i^\theta + \sum_{i=1}^2 e_i \mathcal{O}_i^\eta + \sum_{i=1}^2 s_i \mathcal{O}_i^\phi \right] \quad (69)$$

where p_i , x_i , t_i , e_i and s_i are order one coefficients, $\{\mathcal{O}_i^\varphi, \mathcal{O}_i^{\xi'}, \mathcal{O}_i^\theta, \mathcal{O}_i^\eta, \mathcal{O}_i^\phi\}$ are the complete set of invariant operators of dimension four,

$$\mathcal{O}_1^\varphi = (\varphi^0(\varphi\chi)_{3_2})_{1_2} \theta, \quad \mathcal{O}_2^\varphi = (\varphi^0(\eta\phi)_{3_2})_{1_2} \Delta \quad (70)$$

$$\mathcal{O}_1^\xi = \xi'^0 \theta(\varphi^2)_{1_1}, \quad \mathcal{O}_2^\xi = \xi'^0 \theta(\chi^2)_{1_1}, \quad \mathcal{O}_3^\xi = \xi'^0 \Delta(\eta^2)_{1_1}, \quad \mathcal{O}_4^\xi = \xi'^0 \Delta(\phi^2)_{1_1}, \quad \mathcal{O}_5^\xi = \xi'^0 \Delta^3 \quad (71)$$

$$\mathcal{O}_1^\theta = \theta^0(\eta^3)_{11}, \quad \mathcal{O}_2^\theta = \theta^0(\phi^3)_{11}, \quad \mathcal{O}_3^\theta = \theta^0(\eta\phi^2)_{11} \quad (72)$$

$$\mathcal{O}_1^\eta = (\eta^0(\eta^2)_2)_{12}\theta, \quad \mathcal{O}_2^\eta = (\eta^0(\phi^2)_2)_{12}\theta \quad (73)$$

$$\mathcal{O}_1^\phi = (\phi^0(\phi^2)_{31})_{12}\theta, \quad \mathcal{O}_2^\phi = (\phi^0(\eta\phi)_{31})_{12}\theta \quad (74)$$

The NLO superpotential Δw_v results in shift of the LO VEVs, then the vacuum configuration is modified into

$$\begin{aligned} \langle \varphi \rangle &= (\delta v_{\varphi_1}, v_\varphi + \delta v_{\varphi_2}, \delta v_{\varphi_3}), & \langle \chi \rangle &= (\delta v_{\chi_1}, v_\chi, \delta v_{\chi_3}), \\ \langle \eta \rangle &= (v_\eta, v_\eta + \delta v_{\eta_2}), & \langle \phi \rangle &= (v_\phi + \delta v_{\phi_1}, v_\phi + \delta v_{\phi_2}, v_\phi + \delta v_{\phi_3}), \\ \langle \theta \rangle &= v_\theta + \delta v_\theta \end{aligned} \quad (75)$$

Note that the VEV v_Δ doesn't receive correction at NLO. Similar to section 3, the new vacuum configuration is obtained by imposing the vanish of the first derivative of $w_v + \Delta w_v$ with respect to the driving fields $\varphi^0, \xi^0, \theta^0, \eta^0$ and ϕ^0 . Only terms linear in the shift δv are kept, and terms of order $\delta v/\Lambda$ are neglected, then the minimization equations become,

$$\begin{aligned} (2g_1v_\varphi + g_3v_\chi)\delta v_{\varphi_3} + (2g_2v_\chi - g_3v_\varphi)\delta v_{\chi_3} &= 0 \\ 2g_1\delta v_{\varphi_2} + p_1\frac{v_\chi v_\theta}{\Lambda} &= 0 \\ (2g_1v_\varphi - g_3v_\chi)\delta v_{\varphi_1} + (2g_2v_\chi + g_3v_\varphi)\delta v_{\chi_1} &= 0 \\ g_4(v_\varphi\delta v_{\chi_3} + v_\chi\delta v_{\varphi_3}) + \frac{1}{\Lambda}(2x_3v_\Delta v_\eta^2 + 3x_4v_\Delta v_\phi^2 + x_5v_\Delta^3) &= 0 \\ \kappa v_\theta\delta v_\theta + \frac{1}{\Lambda}(t_1v_\eta^3 + 3t_3v_\eta v_\phi^2) &= 0 \end{aligned} \quad (76)$$

Solving the above linear equations, we obtain

$$\begin{aligned} \delta v_{\varphi_1} &= \frac{2g_2v_\chi + g_3v_\varphi}{g_3v_\chi - 2g_1v_\varphi}\delta v_{\chi_1} \\ \delta v_{\varphi_2} &= -\frac{p_1}{2g_1}\frac{v_\chi v_\theta}{\Lambda} \\ \delta v_{\varphi_3} &= \frac{2g_2v_\chi - g_3v_\varphi}{2g_4\Lambda(2g_1v_\varphi^2 + g_3v_\chi v_\varphi)}[2x_3v_\Delta v_\eta^2 + 3x_4v_\Delta v_\phi^2 + x_5v_\Delta^3] \\ \delta v_{\chi_3} &= -\frac{2g_1v_\varphi + g_3v_\chi}{2g_4\Lambda(2g_1v_\varphi^2 + g_3v_\chi v_\varphi)}[2x_3v_\Delta v_\eta^2 + 3x_4v_\Delta v_\phi^2 + x_5v_\Delta^3] \\ \delta v_\theta &= -\frac{t_1}{\kappa}\frac{v_\eta^3}{\Lambda v_\theta} - \frac{3t_3}{\kappa}\frac{v_\eta v_\phi^2}{\Lambda v_\theta} \end{aligned} \quad (77)$$

where $\delta v_{\varphi_{2,3}}, \delta v_{\chi_3}$ and δv_θ are of order $1/\Lambda$, δv_{χ_1} is undetermined, and it is expected to be suppressed by $1/\Lambda$ as well. The minimization conditions for the shift δv_{η_2} and $\delta v_{\phi_{1,2,3}}$ are

$$\begin{aligned} 2f_2v_\phi(\delta v_{\phi_1} + \delta v_{\phi_2} + \delta v_{\phi_3}) + e_1\frac{v_\eta^2 v_\theta}{\Lambda} + 3e_2\frac{v_\phi^2 v_\theta}{\Lambda} &= 0 \\ 2f_1v_\eta\delta v_{\eta_2} + 2f_2v_\phi(\delta v_{\phi_1} + \delta v_{\phi_2} + \delta v_{\phi_3}) - e_1\frac{v_\eta^2 v_\theta}{\Lambda} - 3e_2\frac{v_\phi^2 v_\theta}{\Lambda} &= 0 \\ f_3(v_\eta\delta v_{\phi_2} - v_\eta\delta v_{\phi_3} - v_\phi\delta v_{\eta_2}) + 2s_2\frac{v_\eta v_\phi v_\theta}{\Lambda} &= 0 \\ f_3(v_\eta\delta v_{\phi_1} - v_\eta\delta v_{\phi_2} - v_\phi\delta v_{\eta_2}) + 2s_2\frac{v_\eta v_\phi v_\theta}{\Lambda} &= 0 \\ f_3(v_\eta\delta v_{\phi_3} - v_\eta\delta v_{\phi_1} - v_\phi\delta v_{\eta_2}) + 2s_2\frac{v_\eta v_\phi v_\theta}{\Lambda} &= 0 \end{aligned} \quad (78)$$

The solutions to the above equations are

$$\begin{aligned}\delta v_{\eta_2} &= \frac{2s_2}{f_3} \frac{v_\eta v_\theta}{\Lambda} \\ \delta v_\phi \equiv \delta v_{\phi_1} = \delta v_{\phi_2} = \delta v_{\phi_3} &= -\frac{e_1}{6f_2} \frac{v_\eta^2 v_\theta}{\Lambda v_\phi} - \frac{e_2}{2f_2} \frac{v_\phi v_\theta}{\Lambda}\end{aligned}\tag{79}$$

We notice that $\langle\phi\rangle$ acquires $\mathcal{O}(1/\Lambda)$ corrections in the same directions, and the shift δv_{η_2} is in general non-zero. Since all the VEVs approximately are of the same order $\mathcal{O}(\lambda_c^2 \Lambda)$ at LO, we expect $\frac{\delta VEV}{VEV} \sim \frac{VEV}{\Lambda} \sim \lambda_c^2$. Therefore the LO vacuum alignment in Eq.(9) and Eq.(12) is stable under the NLO corrections.

References

- [1] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10**, 113011 (2008) [arXiv:0808.2016 [hep-ph]]; M. Maltoni and T. Schwetz, arXiv:0812.3161 [hep-ph].
- [2] M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rept. **460**, 1 (2008) [arXiv:0704.1800 [hep-ph]].
- [3] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002), hep-ph/0202074; P. F. Harrison and W. G. Scott, Phys. Lett. B **535**, 163 (2002), hep-ph/0203209; Z. Z. Xing, Phys. Lett. B **533**, 85 (2002), hep-ph/0204049; X. G. He and A. Zee, Phys. Lett. B **560**, 87 (2003), hep-ph/0301092.
- [4] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [5] C. Amsler et al. (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
- [6] S. F. King, JHEP **0508** (2005) 105 [arXiv:hep-ph/0506297]; I. de Medeiros Varzielas and G. G. Ross, arXiv:hep-ph/0507176; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **644** (2007) 153 [arXiv:hep-ph/0512313]; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **648** (2007) 201 [arXiv:hep-ph/0607045]; S. F. King and M. Malinsky, JHEP **0611** (2006) 071 [arXiv:hep-ph/0608021]; F. Feruglio and Y. Lin, Nucl. Phys. B **800**, 77 (2008) [arXiv:0712.1528 [hep-ph]]. .
- [7] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [arXiv:hep-ph/9310320]; M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **398**, 319 (1993) [arXiv:hep-ph/9212278]; L. E. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994), arXiv:hep-ph/9403338; P. Binetruy and P. Ramond, Phys. Lett. B **350**, 49 (1995), arXiv:hep-ph/9412385; E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **356**, 45 (1995), arXiv:hep-ph/9504292.
- [8] G. G. Ross, "Models of fermions masses", *Prepared for Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 2000): Flavor Physics for the Millennium, Boulder, Colorado, 4-30 Jun 2000*; G. Altarelli, arXiv:0711.0161 [hep-ph].
- [9] E. Ma and G. Rajasekaran, Phys. Rev. D **64** (2001) 113012, arXiv:hep-ph/0106291; E. Ma, Mod. Phys. Lett. A **17** (2002) 627 [arXiv:hep-ph/0203238]; K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552** (2003) 207, arXiv:hep-ph/0206292; M. Hirsch, J. C. Romao, S. Skadhauge, J. W. F. Valle and A. Villanova del Moral, arXiv:hep-ph/0312244; Phys. Rev. D **69** (2004) 093006 [arXiv:hep-ph/0312265]; E. Ma, Phys. Rev. D **70** (2004) 031901; Phys. Rev. D **70** (2004) 031901 [arXiv:hep-ph/0404199]; New J. Phys. **6** (2004) 104 [arXiv:hep-ph/0405152]; arXiv:hep-ph/0409075; S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B **724** (2005) 423 [arXiv:hep-ph/0504181]; E. Ma, Phys. Rev. D **72** (2005) 037301 [arXiv:hep-ph/0505209]; M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma, Phys. Rev. D **72** (2005) 091301 [Erratum-ibid. D **72** (2005) 119904] [arXiv:hep-ph/0507148]. K. S. Babu and X. G. He, arXiv:hep-ph/0507217; E. Ma, Mod. Phys. Lett. A **20** (2005) 2601 [arXiv:hep-ph/0508099]; A. Zee, Phys.

Lett. B **630** (2005) 58 [arXiv:hep-ph/0508278]; E. Ma, Phys. Rev. D **73** (2006) 057304 [arXiv:hep-ph/0511133]; X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **0604** (2006) 039 [arXiv:hep-ph/0601001]; B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M. K. Parida, Phys. Lett. B **638** (2006) 345 [arXiv:hep-ph/0603059]; E. Ma, Mod. Phys. Lett. A **21** (2006) 2931 [arXiv:hep-ph/0607190]; Mod. Phys. Lett. A **22** (2007) 101 [arXiv:hep-ph/0610342]; L. Lavoura and H. Kuhbock, Mod. Phys. Lett. A **22** (2007) 181 [arXiv:hep-ph/0610050]; E. Ma, H. Sawanaka and M. Tanimoto, Phys. Lett. B **641**, 301 (2006), hep-ph/0606103; S. F. King and M. Malinsky, Phys. Lett. B **645** (2007) 351 [arXiv:hep-ph/0610250]; S. Morisi, M. Picariello and E. Torrente-Lujan, Phys. Rev. D **75** (2007) 075015 [arXiv:hep-ph/0702034]; M. Hirsch, A. S. Joshipura, S. Kaneko and J. W. F. Valle, Phys. Rev. Lett. **99**, 151802 (2007) [arXiv:hep-ph/0703046]. F. Yin, Phys. Rev. D **75** (2007) 073010 [arXiv:0704.3827 [hep-ph]]; F. Bazzocchi, S. Kaneko and S. Morisi, JHEP **0803** (2008) 063 [arXiv:0707.3032 [hep-ph]]; F. Bazzocchi, S. Morisi and M. Picariello, Phys. Lett. B **659** (2008) 628 [arXiv:0710.2928 [hep-ph]]; M. Honda and M. Tanimoto, Prog. Theor. Phys. **119** (2008) 583 [arXiv:0801.0181 [hep-ph]]; B. Brahmachari, S. Choubey and M. Mitra, Phys. Rev. D **77** (2008) 073008 [Erratum-ibid. D **77** (2008) 119901] [arXiv:0801.3554 [hep-ph]]; F. Bazzocchi, S. Morisi, M. Picariello and E. Torrente-Lujan, J. Phys. G **36** (2009) 015002 [arXiv:0802.1693 [hep-ph]]; B. Adhikary and A. Ghosal, Phys. Rev. D **78** (2008) 073007 [arXiv:0803.3582 [hep-ph]]; M. Hirsch, S. Morisi and J. W. F. Valle, Phys. Rev. D **78** (2008) 093007 [arXiv:0804.1521 [hep-ph]]; Y. Lin, Nucl. Phys. B **813**, 91 (2009) [arXiv:0804.2867 [hep-ph]]; P. H. Frampton and S. Matsuzaki, arXiv:0806.4592 [hep-ph]; F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B **809**, 218 (2009) [arXiv:0807.3160 [hep-ph]]; F. Bazzocchi, M. Frigerio and S. Morisi, Phys. Rev. D **78**, 116018 (2008) [arXiv:0809.3573 [hep-ph]]; S. Morisi, Phys. Rev. D **79**, 033008 (2009) [arXiv:0901.1080 [hep-ph]]; P. Ciafaloni, M. Picariello, E. Torrente-Lujan and A. Urbano, Phys. Rev. D **79**, 116010 (2009) [arXiv:0901.2236 [hep-ph]]; M. C. Chen and S. F. King, JHEP **0906**, 072 (2009) [arXiv:0903.0125 [hep-ph]]; T. J. Burrows and S. F. King, arXiv:0909.1433 [hep-ph]. .

- [10] G. Altarelli and F. Feruglio, Nucl. Phys. B **720**, 64 (2005), hep-ph/0504165.
- [11] G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006), hep-ph/0512103.
- [12] G. Altarelli, F. Feruglio and C. Hagedorn, JHEP **0803**, 052 (2008) [arXiv:0802.0090 [hep-ph]].
- [13] G. C. Branco, R. Gonzalez Felipe, M. N. Rebelo and H. Serodio, Phys. Rev. D **79**, 093008 (2009) [arXiv:0904.3076 [hep-ph]].
- [14] G. Altarelli and D. Meloni, J. Phys. G **36**, 085005 (2009) [arXiv:0905.0620 [hep-ph]].
- [15] E. Bertuzzo, P. Di Bari, F. Feruglio and E. Nardi, arXiv:0908.0161 [hep-ph].
- [16] C. Hagedorn, E. Molinaro and S. T. Petcov, JHEP **0909**, 115 (2009) [arXiv:0908.0240 [hep-ph]].

- [17] D. Aristizabal Sierra, F. Bazzocchi, I. de Medeiros Varzielas, L. Merlo and S. Morisi, arXiv:0908.0907 [hep-ph].
- [18] R. G. Felipe and H. Serodio, arXiv:0908.2947 [hep-ph].
- [19] G. J. Ding, Phys. Rev. D **78**, 036011 (2008) [arXiv:0803.2278 [hep-ph]].
- [20] P. D. Carr and P. H. Frampton, arXiv:hep-ph/0701034.
- [21] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B **775**, 120 (2007), hep-ph/0702194.
- [22] M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007), arXiv:0705.0714 [hep-ph].
- [23] P. H. Frampton and T. W. Kephart, JHEP **0709**, 110 (2007), arXiv:0706.1186 [hep-ph].
- [24] A. Aranda, Phys. Rev. D **76**, 111301 (2007), arXiv:0707.3661 [hep-ph].
- [25] P. H. Frampton, T. W. Kephart and S. Matsuzaki, Phys. Rev. D **78**, 073004 (2008) [arXiv:0807.4713 [hep-ph]].
- [26] D. A. Eby, P. H. Frampton and S. Matsuzaki, Phys. Lett. B **671**, 386 (2009) [arXiv:0810.4899 [hep-ph]].
- [27] E. Ma, Phys. Lett. B **632**, 352 (2006) [arXiv:hep-ph/0508231].
- [28] F. Bazzocchi and S. Morisi, arXiv:0811.0345 [hep-ph].
- [29] F. Bazzocchi, L. Merlo and S. Morisi, arXiv:0901.2086 [hep-ph].
- [30] F. Bazzocchi, L. Merlo and S. Morisi, arXiv:0902.2849 [hep-ph].
- [31] H. Ishimori, Y. Shimizu and M. Tanimoto, Prog. Theor. Phys. **121**, 769 (2009) [arXiv:0812.5031 [hep-ph]].
- [32] G. Altarelli, F. Feruglio and L. Merlo, JHEP **0905**, 020 (2009) [arXiv:0903.1940 [hep-ph]].
- [33] S. Pakvasa and H. Sugawara, Phys. Lett. B **82**, 105 (1979); T. Brown, N. Deshpande, S. Pakvasa and H. Sugawara, Phys. Lett. B **141**, 95 (1984); Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. D **25**, 1895 (1982) [Erratum-ibid. D **29**, 2135 (1984)]; T. Brown, S. Pakvasa, H. Sugawara and Y. Yamanaka, Phys. Rev. D **30**, 255 (1984).
- [34] D. G. Lee and R. N. Mohapatra, Phys. Lett. B **329**, 463 (1994) [arXiv:hep-ph/9403201]; C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP **0606**, 042 (2006) [arXiv:hep-ph/0602244]; Y. Cai and H. B. Yu, Phys. Rev. D **74**, 115005 (2006) [arXiv:hep-ph/0608022]; H. Zhang, Phys. Lett. B **655**, 132 (2007) [arXiv:hep-ph/0612214]; Y. Koide, JHEP **0708**, 086 (2007) [arXiv:0705.2275 [hep-ph]]; M. K. Parida, Phys. Rev. D **78**, 053004 (2008) [arXiv:0804.4571 [hep-ph]].
- [35] C. S. Lam, Phys. Rev. Lett. **101**, 121602 (2008) [arXiv:0804.2622 [hep-ph]].

- [36] C. S. Lam, *Phys. Rev. D* **78**, 073015 (2008) [arXiv:0809.1185 [hep-ph]].
- [37] C. S. Lam, arXiv:0907.2206 [hep-ph].
- [38] W. Grimus, L. Lavoura and P. O. Ludl, arXiv:0906.2689 [hep-ph].
- [39] P. Minkowski, *Phys. Lett.* **B67** (1977) 421; T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979); S. L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980); R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44** (1980) 912.
- [40] C. D. Froggatt and H. B. Nielsen, *Nucl. Phys. B* **147**, 277 (1979).
- [41] A. Osipowicz *et al.* [KATRIN Collaboration], arXiv:hep-ex/0109033; see also: <http://www-ik.fzk.de/~katrin/index.html>
- [42] L. Baudis *et al.*, *Phys. Rev. Lett.* **83**, 41 (1999) [arXiv:hep-ex/9902014].
- [43] A. Giuliani [CUORE Collaboration], *J. Phys. Conf. Ser.* **120** (2008) 052051.
- [44] Majorana Collaboration, arXiv:0811.2446 [nucl-ex].
- [45] A. A. Smolnikov and f. t. G. Collaboration, arXiv:0812.4194 [nucl-ex].
- [46] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, arXiv:0809.2936 [hep-ph].
- [47] WMAP Collaboration, E. Komatsu *et al.*, arXiv:0803.0547 [astro-ph].
- [48] ACBAR Collaboration, C. L. Reichardt *et al.*, arXiv:0801.1491 [astro-ph].
- [49] VSA Collaboration, C. Dickinson *et al.*, *Mon. Not. Roy. Astron. Soc.* **353**, 732 (2004) [arXiv:astro-ph/0402498].
- [50] CBI Collaboration, A. C. S. Readhead *et al.*, *Astrophys. J.* **609**, 498 (2004) [arXiv:astro-ph/0402359].
- [51] BOOMERANG Collaboration, C. J. MacTavish *et al.*, *Astrophys. J.* **647**, 799 (2006) [arXiv:astro-ph/0507503].
- [52] SDSS Collaboration, M. Tegmark *et al.*, *Phys. Rev. D* **74** (2006) 123507 [arXiv:astro-ph/0608632].
- [53] SNLS Collaboration, P. Astier *et al.* *Astron. Astrophys.* **447**, 31 (2006) [arXiv:astro-ph/0510447].
- [54] SDSS Collaboration, D. J. Eisenstein *et al.*, *Astrophys. J.* **633**, 560 (2005) [arXiv:astro-ph/0501171].

- [55] P. McDonald *et al.*, *Astrophys. J. Suppl.* **163**, 80 (2006); P. McDonald *et al.*, *Astrophys. J.* **635**, 761 (2005).
- [56] A. Pilaftsis and T. E. J. Underwood, *Phys. Rev. D* **72**, 113001 (2005) [arXiv:hep-ph/0506107].
- [57] Gui-Jun Ding, work in progress.
- [58] F. Ardellier *et al.* [Double Chooz Collaboration], arXiv:hep-ex/0606025.
- [59] Y. f. Wang, arXiv:hep-ex/0610024.